# PROCEEDINGS

OF THE

# AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 71 OCTOBER, 1945 No. 8

## TECHNICAL PAPERS

AND

## DISCUSSIONS

Published monthly, except July and August, at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York 18, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society are given.

Entered as Second-Class Matter, September 23, 1937, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum. Price $1.00 per copy

*Copyright, 1945, by the* AMERICAN SOCIETY OF CIVIL ENGINEERS

*Printed in the United States of America*

| | Published | Discussion closes |
|---|---|---|
| *Johnson, Lee H., Jr.* Analysis of Statically Indeterminate Structures Using Reduced Equations | Nov., 1944 | |
| Discussion in Feb., May, June, 1945 | | Closed* |
| *Kalinske, A. A.* Conversion of Kinetic to Potential Energy in Flow Expansions | Dec., 1944 | |
| Discussion in Feb., May, June, Sept., 1945 | | Closed* |
| *DeYoung, Isaac.* A Plan for a Movable Dam | Feb., 1945 | |
| Discussion in May, June, Oct., 1945 | | Closed* |
| *Conkling, Harold.* Utilization of Ground-Water Storage in Stream System Development | Jan., 1945 | |
| Discussion in Mar., Apr., May, June, Sept., 1945 | | Closed* |
| *Bull, Anders.* Stresses in the Linings of Shield-Driven Tunnels | Nov., 1944 | |
| Discussion in Jan., Feb., May, June, Sept., 1945 | | Nov. 1, 1945 |
| *Osgood, William R.* Column Formulas | Dec., 1944 | |
| Discussion in Feb., Mar., May, June, 1945 | | Nov. 1, 1945 |
| *Bennett, Preston T.* The Effect of Blankets on Seepage Through Pervious Foundations | Jan., 1945 | |
| Discussions in Mar., Apr., June, Oct., 1945 | | Nov. 1, 1945 |
| *Weiss, Camillo.* Interrelation of Certain Structural Concepts | Jan., 1945 | |
| Discussion in May, June, Sept., Oct., 1945 | | Nov. 1, 1945 |
| *Blanks, R. F., and Meissner, H. S.* Deterioration of Concrete Dams Due to Alkali-Aggregate Reaction | Jan., 1945 | |
| Discussion in Sept., 1945 | | Dec. 1, 1945 |
| *Powell, Ralph W.* Flow in a Channel of Definite Roughness | Dec., 1944 | |
| Discussion in June, Oct., 1945 | | Dec. 1, 1945 |
| *Bakhmeteff, Boris A., and Allan, William.* The Mechanism of Energy Loss in Fluid Friction | Feb., 1945 | |
| Discussion in Sept., 1945 | | Jan. 1, 1946 |
| *Hayes, Harry.* Elimination of Cross Connections in Los Angeles, Calif. | Mar., 1945 | |
| Discussion in May, Oct., 1945 | | Jan. 1, 1946 |
| *Eliel, Leon T.* Economics of Photogrammetry | Mar., 1945 | |
| Discussion in June, Oct., 1945 | | Jan. 1, 1946 |
| *Nelidov, I. M.* Determination of Pressures within a Siphon Spillway | Mar., 1945 | Jan. 1, 1946 |
| *Genter, A. L.* Computing Coagulant Requirements in Sludge Conditioning | Mar., 1945 | |
| Discussion in June, Oct., 1945 | | Jan. 1, 1946 |
| *Mullis, Ira B.* Design of Roadbeds | Apr., 1945 | |
| Discussion in Oct., 1945 | | Jan. 1, 1946 |
| *Harrold, Lloyd L.* Flow from Drainage Basins Determined by Short-Term Records | Apr., 1945 | |
| Discussion in Oct., 1945 | | Jan. 1, 1946 |
| *Camp, Thomas R.* Sedimentation and the Design of Settling Tanks | Apr., 1945 | |
| Discussion in September, 1945 | | Jan. 1, 1946 |
| *Stevens, J. C.* Future of Lake Mead and Elephant Butte Reservoir | May, 1945 | |
| Discussion in Oct., 1945 | | Jan. 1, 1946 |
| *Brumfield, R. C.* Moving Loads on Restrained Beams and Frames | May, 1945 | |
| Discussion in Oct., 1945 | | Jan. 1, 1946 |
| *Eichler, John Oran, and Eves, Howard W.* Mathematical Examination of the Highway Transition Spiral | May, 1945 | Jan. 1, 1946 |
| *Report:* Water Supply Engineering: Report of the Committee of the Sanitary Engineering Division for the Two Years Ending December 31, 1944 | May, 1945 | Jan. 1, 1946 |
| *Johnston, Milan A.* Effects of Radiant Heat on Reinforced-Concrete Rigid Frames | June, 1945 | |
| Discussion in Oct., 1945 | | Jan. 1, 1946 |
| *Ehasz, F. L.* Structural Skew Plates | June, 1945 | Jan. 1, 1946 |
| *Laverty, Finley B.* Correlating Flood Control and Water Supply, Los Angeles Coastal Plain | June, 1945 | Jan. 1, 1946 |
| *Symposium:* Cavitation in Hydraulic Structures | Sept., 1945 | Feb. 1, 1946 |

NOTE.—The closing dates herein published are final except when names of prospective discussers are registered for special extension of time.

* Publication of closing discussion pending.

# CONTENTS FOR OCTOBER, 1945

## PAPERS

PAGE

The Safety of Structures.
*By Alfred M. Freudenthal* ..... 1157

Design Developments—Structures of the Tennessee Valley Authority: A Symposium ..... 1193

## DISCUSSIONS

The Effect of Blankets on Seepage Through Pervious Foundations.
*By W. J. Turnbull, and Kenneth S. Lane* ..... 1233

A Plan for a Movable Dam.
*By L. C. Sabin, and A. W. Sargent* ..... 1245

Moving Loads on Restrained Beams and Frames.
*By R. W. Stewart, Nathan D. Brodkin, and T. F. Hickerson* ..... 1251

Flow in a Channel of Definite Roughness.
*By Garbis H. Keulegan* ..... 1259

Future of Lake Mead and Elephant Butte Reservoir.
*By D. C. Bondurant, John H. Bliss, Luna B. Leopold, Carl B. Brown, and G. E. P. Smith* ..... 1263

Economics of Photogrammetry.
*By George D. Whitmore, Benjamin A. Wasil, Charles H. Davey, and Robert H. Randall* ..... 1282

Computing Coagulant Requirements in Sludge Conditioning.
*By D. L. McLean, and K. Fraschina* ..... 1289

Effects of Radiant Heat on Reinforced-Concrete Rigid Frames.
*By I. Oesterblom* ..... 1295

Interrelation of Certain Structural Concepts.
*By T. F. Hickerson* ..... 1301

# CONTENTS FOR OCTOBER, 1945 (*Continued*)

PAGE

Elimination of Cross Connections in Los Angeles, Calif.
*By Robert P. Lowe* ..... 1305

Design of Roadbeds.
*By Charles W. Britzius, J. T. L. McNew, and Gustavo Pérez Guerra* ..... 1307

Flow from Drainage Basins Determined by Short-Term Records.
*By Samuel Roberts* ..... 1311

---

*A list of "Current Papers and Discussions" may be found on the page preceding the table of contents*

*The Society is not responsible for any statement made or opinion expressed in its publications*

## *HINTS TO AUTHORS*

*Those who are planning papers for submission to* PROCEEDINGS *will expedite Committee action measurably by first studying the standard instructions as to style, content, and format. For reprints, address the Secretary, Committee on Publications, 33 W. 39th St., New York 18, N. Y.*

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

# THE SAFETY OF STRUCTURES

By Alfred M. Freudenthal,[1] Assoc. M. Am. Soc. C. E.

Synopsis

The purpose of this paper is to analyze the safety factor in engineering structures in order to establish a rational method of evaluating its magnitude. The discrepancy between the highly refined procedure of modern design and the rather arbitrary manner of choosing the safety factor is seriously hampering the development of more effective design methods based upon a perfect balance of safety and economy.

The true character of the safety factor is disclosed by the introduction of a statistical concept of physical qualities, according to which the individual properties composing the structural phenomena of strain and resistance are represented by frequency distributions instead of by individual (minimum or maximum) values. The safety factor, correlating the strain induced with the resistance of the structure, may be derived from observable and measurable physical properties and phenomena.

The correlation of strain and resistance requires a careful analysis of all features of structural design, both from the points of view of the basic assumptions and of statistical interpretation. For the purpose of this analysis the available experimental and observational evidence is rather inadequate; the results obtainable at present, therefore, should be considered suggestive rather than conclusive. It is the insight gained into the essence of the safety factor that is of immediate practical interest.

The word "strain" in this paper is not to be confused with its conventional use to denote a linear measure, or proportion of stretch. It has a meaning roughly equivalent to what is commonly called "stress," but is more general as it is intended to denote the mechanical effect of load, or external conditions on the structure, the structural member, or the section. The word "resistance"

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **March 1, 1946.**

[1] Lecturer in Bridge Eng., Hebrew Inst. of Tech., Haifa, Palestine.

is a general term indicating "strength" or capacity to resist failure. It can be expressed in any units appropriate to the matter under discussion.

## PART 1.—GENERAL PRINCIPLES

### Introduction

Structural design is generally understood to be composed of two operations—(1) The determination of the strain induced in the structure by loads and secondary effects, and (2) the comparison of the maximum strain that occurs with a rather fictitious resistance characteristic termed the "permissible stress," which is a fraction of the strength of the respective structural material. Although attempts have been made to modify this latter concept,[2,3] which is theoretically inadequate for the rational solution of any but the most simple structural problems, such new ways have not succeeded in affecting the practical procedure of design.

Structural design, like technical problems in general, must not be considered solely from the point of view of theoretical accuracy detached from the aspect of practical application. The conventional methods have proved their expediency under widely differing conditions. Although the underlying concepts have lost their original significance as a consequence of the better understanding of the behavior of structures and structural materials, it appears reasonable to retain the well-tested traditional form and to adapt it to the present improved knowledge of the problem, by redefining the basic concepts in accordance with the results of advanced research.

The fundamental, conventional concept of "allowable stress" involves a comparison between a computed maximum strain and the strength of the material, and implies the existence of a margin between the two. The justification of this margin has never been contested. As its conventional name "margin of safety" suggests, it reveals the subjective striving on the part of the designer for an adequate measure of safety as well as a consciousness of the limitations of his knowledge and the arbitrariness of his assumptions. Its real character has remained obscure, however, and its magnitude is generally estimated on the basis of subjective judgment rather than objective fact. Nothing has been done to establish a criterion for its determination on a more rational basis than the experience and judgment of the designer. The most refined design is thus deprived of its merits, the designer being free to choose the fundamental assumptions of his design largely on the basis of subjective arguments, without being compelled to ascertain their validity by the identification of the objective conditions. Research in the sphere of new materials cannot be expected to bear its full weight upon the economy of structures if the safety factor can be fixed rather arbitrarily "between 1.25 and 4.00 or even higher."[4] Therefore, the analysis of the safety factor to identify its true character and to determine its objective values has become of increasing urgency.

[2] *Final Report*, 2d Cong. of the International Assn. for Bridge and Structural Eng., 1936, Pt. I, Ductility of Steel, Wilhelm Ernst & Sohn, Berlin, 1939, p. 27.

[3] "Theory of Limit Design," by J. A. Van den Broek, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), pp. 638–730.

[4] "Structural Application of Steel and Light-Weight Alloys," A Symposium, *ibid.*, Vol. 102 (1937), pp. 1207–1208.

## The Factor of Safety

The principle underlying the concept of the safety factor can be understood best by reviewing the fundamental difficulty of structural design. The computed structural characteristic ("strain") cannot be equated to the characteristic derived from observable and measurable physical properties ("resistance") because the results of an intellectual process—design—cannot be equated to the result of material perceptions—resistance. The limitations of human observation are such that no observable quality can be measured "exactly." The designer is able to assert only that a value is larger than a lower limit and smaller than an upper limit—not that it is "equal" to another value. The manifest lack of correspondence between the conceived action and the performed action in any sphere of human activity explains why no real identity may be expected in a series of actions or events which were planned to be identical. Consequently, intellectual concepts, contrived to reproduce material phenomena with a certain grade of perfection, may be correlated with material observations regarding those phenomena only by a relation of "inequality," expressed in terms of a probable correlation range. This range itself will be a function of the degree of perfection in the concept. It must therefore provide for: (1) The imperfection of human observations and actions (uncertainty); and (2) the imperfections of intellectual concepts devised to reproduce physical phenomena (ignorance). This range represents the objective minimum value of the safety margin, which is thus identified as a function of objective uncertainty as well as subjective ignorance.

With increasing perfection of design methods the element of "ignorance" can be largely eliminated; but the element of "uncertainty" is caused by circumstances that can be changed, to a certain extent, but can never be removed. Hence, the safety factor is a measure of uncertainty rather than of ignorance. The trend toward reducing its numerical value is not so much the result of improved design methods as it is the result of modified objective circumstances; that is, standardizing engineering materials by introducing quality control in production, applying standard acceptance tests by the users of such materials, and introducing stringent regulations for the control of workmanship.

The laws of structural design are derived from the principles of classical mechanics, and are based on the existence of a causal relationship between the antecedent and the consequent events. They are mostly expressed in the form of differential equations, the solution of which enables the engineer to determine all the consequences following one or a number of given antecedent events. Within the range for which the initial assumptions are valid, therefore, the designer should be able to go confidently from cause to effect, all phenomena concerned being strictly predictable. A certain number of parameters of these equations represent observable and measurable physical properties or phenomena. The application of the differential equations to structural design requires the introduction of the real values of such properties under all conceivable conditions of practical importance. Some of these values must be predicted or estimated since their observation and measurement under all relevant conditions are impracticable. Such prediction is entirely different from that based

upon differential equations. It can only refer to past experience; and the effect of chance, inherent in any activity or perception, becomes essential. The concept of deterministic causality is superseded by a new concept, in which every unknown cause is termed a "chance" cause. Systems of chance causes produce events in accordance with the law of large numbers and thus give rise to statistical laws represented by frequency distributions. Prediction based on a statistical causality, therefore, may be expressed only in terms of the probability that a certain event will occur within definite limits.

With the introduction of statistical causality the conventional conceptions of physical qualities undergo a considerable change. Every physical property is a statistical distribution so that the constancy of physical qualities is found to be of a purely statistical nature. A given quality approaches a constant value only in the sense that it may be represented by a frequency distribution within specified limits. The laws of structural design, therefore, must be considered a combination of functional and statistical relationships—functional so far as the laws of the theory of structures are concerned and statistical to the extent that real physical properties appear as parameters of the functional relations.

There are cases in which apparently functional relations are intrinsically statistic. Relationships that are important in a practical sense frequently cannot be derived from differential equations either because such equations are not known for the given case or because their solution is too difficult. The laws derived in such cases by statistical interpretation of past experience are empiric and do not represent real functional relationships. They represent the trend revealed by the correlation of assumedly relevant variables; the pertinent frequency distributions form an integral part of the information.

The interrelation between buckling resistance $s_b$ and the slenderness ratio $\frac{l}{r}$ of steel columns is an example of the simultaneous existence of functional, empirical, and statistical laws in one and the same problem. Fig. 1 affords an illustration of the relation—

$$s_b = f\left(\frac{l}{r}\right) \qquad (1)$$

Region c defines Euler's hyperbola as derived from the differential equation of elastic buckling; region b represents the empirical laws of inelastic buckling; and region a shows the range of pure compression in which the frequency distribution of the resistance is the effect of a system of genuine chance causes only.

The value of the safety factor $\zeta$ may be derived from the condition that the maximum strain $s_a$ induced in the structure by actual service conditions must never cause such damage as to impede its fitness for service, even were this maximum strain to coincide with the lowest value of the structure's resistance $s_r$

$$\zeta = \frac{s_r}{s_a} > 1 \qquad (2)$$

If $\pm \Delta s_a$ denotes the maximum range of fluctuation of actual strain about the expected value $s_{ao}$ and $\pm \Delta s_r$ the maximum range of fluctuation of the

structure's resistance about its expected value $s_{ro}$ the maximum strain to be expected will be $(s_{ao} + \Delta s_a)$ and the minimum resistance will never be less than $s_{ro} - \Delta s_{ro}$. According to Eq. 2 failure is just prevented if $s_r$ (minimum) $= s_a$ (maximum) or

$$s_{ao} + \Delta s_a = s_{ro} - \Delta s_r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Eq. 3 leads to the condition of minimum safety based on the correlation of

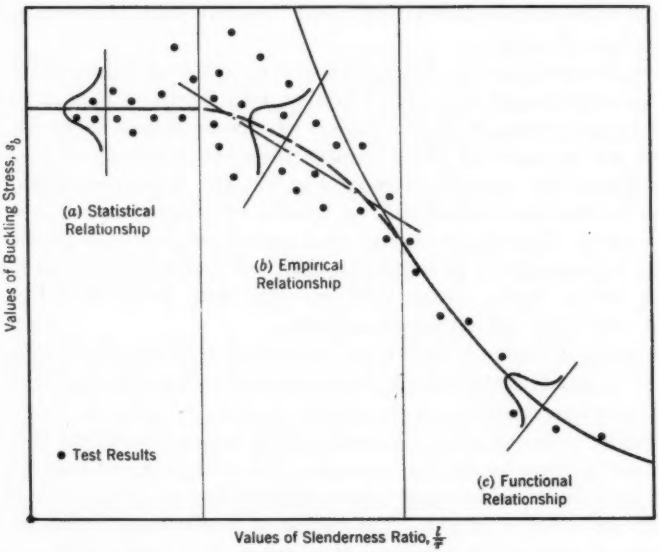



FIG. 1.—BUCKLING STRESS $s_b = f(l/r)$ OF STEEL COLUMNS

expected values of strain and resistance; thus:

$$s_{ao} = \frac{1 - \dfrac{\Delta s_r}{s_{ro}}}{1 + \dfrac{\Delta s_a}{s_{ao}}} s_{ro} = \frac{1}{\zeta} s_{ro} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which,

$$\zeta = \frac{1 + \dfrac{\Delta s_a}{s_{ao}}}{1 - \dfrac{\Delta s_r}{s_{ro}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

denotes the safety factor. For convenience, the correlation of strain and resistance may be expressed as a correlation of load and carrying capacity.

## ABSOLUTE VERSUS RELATIVE, OR "ECONOMIC," SAFETY

The value of the safety factor derived from the condition of absolute safety expressed by Eq. 2 depends upon the variability of the design parameters (dimensions, weights, elasticity, stength, temperature) and of the initial and boundary conditions. The individual ranges of fluctuation determine the

ultimate ranges of variation of strain and resistance or of load and carrying capacity, in accordance with the law of statistical superposition. An increase in the individual ranges as well as in the number of parameters increases the resulting factor of safety. In order to keep this factor as low as is reasonably possible it is necessary to reduce the range of dispersion of the individual parameter by: (*a*) Modifying the objective circumstances and conditions under which it operates until the range of variability of the parameters of the design is that of genuine chance fluctuations; and (*b*) introducing the economic aspect of the problem of safety.

The requirement that the fluctuation of the individual values of the relevant material properties must be confined to the range of genuine chance fluctuation, produced by a system of large numbers of chance causes in which any one cause does not produce an effect greater than the resultant effect of all the others, expresses the purpose of control in modern industrial production and represents its standard definition.[5] A quality or a phenomenon is said to be controlled when, through the use of past experience, one can predict, within limits, how this quality or phenomenon may be expected to vary in the future. Prediction within limits means that one can state the probability that an individual value will fall within given limits.

By enforcing a régime of maximum control upon all material properties and phenomena that affect the design, eliminating all assignable causes of fluctuation, the objective minimum value of the factor of "absolute" safety is obtained. Such "absolute" safety is economically impracticable and theoretically meaningless. Failure cannot be prevented with certainty but only with a high degree of probability, which may approach certainty (a probability of 1) very closely but can never attain it. The ranges of fluctuation (which tend to become infinite to insure certainty) are thus reduced to a reasonable magnitude. A factor of safety may be based on the probability that individual values of either strain or resistance will fall beyond the ranges of either

$$\zeta = s_{ao} + j\,\sigma_s \qquad (6a)$$

or

$$\zeta = s_{ro} - j\,\sigma_r \qquad (6b)$$

in which $\sigma_s$ and $\sigma_r$ denote the standard deviation of the universes of $s_a$ and $s_r$, respectively, and $j$ is an arbitrary constant. Failure can be made highly improbable if this probability is small enough, but cannot be prevented absolutely.

The degree of safety will depend on the selection of the values of the coefficient $j$, resulting from a reasonable compromise between the requirements of safety and those of economy. It will be comparatively easy to reach such a compromise if the entire process of construction, beginning with the manufacture of the structural material, has been under rigorous control. If a certain level of economy is to be maintained, deficiency of control must be compensated by an increase in the risk faced by the designer.

[5] "Economic Control of Quality of Manufactured Product," by W. A. Shewart, D. Van Nostrand Co., Inc., New York, N. Y., 1931, p. 151.

### Prediction from Past Experience

A structure is designed by predicting its future behavior on the strength of knowledge gained by past experience and the analysis of factual data. The computation of its safety factor requires an analysis of the variability of all influences bearing upon its resistance and the strain imposed. The quantitative information available is inadequate; where data have actually been collected they are usually presented inadequately and are thus deprived of much of their value.

Past experience concerning physical properties is generally recorded as a series of observations of the properties themselves, or of quality characteristics expressing a known relationship with the property considered. This latter relationship may be either functional (as in observing the decreasing amplitude of vibrations to determine the coefficient of damping of a material) or empirical (as in deriving the tensile strength of steel by observing the hardness of its surface).

When, in the case of some complex phenomena, research has not yet resulted in the establishment of such relationships as would be required for the resolution of the phenomenon into its fundamental components, past experience must be trusted to a correspondingly greater degree. Then past experience may be expressed either by series of observations or measurements of a characteristic manifestation of the phenomenon, or, if such evidence is inconclusive, by the ultimate limits within which all values of the characteristic may be expected to lie. Such limits will frequently be deducible from general considerations, as, for example, in delimiting the possible extreme fluctuations in the bending resistance of sections of mild steel between the resistance of an entirely plastic section with a rectangular stress distribution and that of a maximum elastic section with a triangular stress distribution.[2] Sometimes the frequency distribution itself (and thus the probability of the occurrence) of an individual event can be predicted from the knowledge of its a priori probability, which is the objective probability that an event will occur in an infinite number of trials. This probability can generally be derived by inference from collateral information at the disposal of the designer.

When dealing with material properties such as strength, elasticity, or linear dimensions—all of which affect the resistance of the structure—it will be necessary to predict their most probable values and the probable ranges of fluctuations of individual values. Observations of only a comparatively small number of samples, supplemented by relevant knowledge based on experience with the manufacture and the procedure of control and selection will be available for this purpose. The character of that process and the extent and reliability of current knowledge concerning it will determine the character and shape of the distribution function of the property considered, and therefore the accuracy of the prediction.

When dealing with external influences, such as service loads or wind pressure, that affect the strain induced in the structure the designer will have to assess the probability that certain extremes will occur; or he will be required to estimate the most probable conditions and probable ranges of fluctuations

from objective a priori probabilities based, generally, not on direct statistical evidence (which is mostly not available) but on circumstantial evidence. The form of the distribution curve of the probabilities of occurrence of certain loading conditions may be determined by the law of large numbers. For example, the probability that a certain load will occur $m$ times in $n$ cases is given by the binomial, which is the $m$th successive term of the expression $(P_p + P_q)^n$, in which $P_p$ denotes the a priori or mathematical probability of the occurrence of the particular load and $P_q = (1 - P_p)$ the probability that it will not occur.

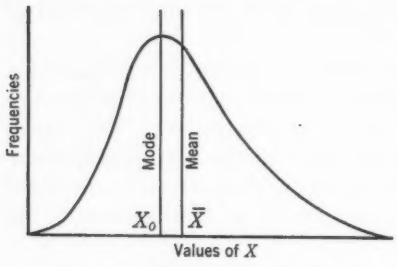



FIG. 2.—FREQUENCY DISTRIBUTION OF CHARACTERISTIC $X$

In order that data and statistical information may be used effectively, they will have to be condensed and presented in the form of a frequency distribution. Subsequently, the frequency distribution should be approximated by an algebraic distribution function $f(X, \bar{X}, \sigma, k)$ of the respective quality characteristic $X$ (Fig. 2), containing the three principal statistics of the frequency distribution computed from the $n$ observations $X_n$ of the characteristic $X$; namely, its expected or mean value

$$\bar{X} = \frac{\Sigma X_n}{n} \qquad (7a)$$

its standard deviation

$$\sigma = \sqrt{\frac{\Sigma(X_n - \bar{X})^2}{n}} \qquad (7b)$$

and its skewness

$$k = \frac{\Sigma(X_n - \bar{X})^3}{n\,\sigma^3} \qquad (7c)$$

If the parameters of the distribution function are chosen so that

$$\int_{-\infty}^{+\infty} f(X, \bar{X}, \sigma, k)\, dX = 1 \qquad (8)$$

the integral $\int_a^b f(X, \bar{X}, \sigma, k)\, dX$ is a measure of the probability that an individual value $X_n$ will occur within the limits $X = a$ and $X = b$.

The shape of the frequency distribution of a material property is an indication of the conditions under which this property has been, or is being, produced. In manufacturing processes an effort is generally made to attain a definite value of the characteristic property. This objective can be expressed either by stipulating a minimum value of the property or by imposing a limit on the fluctuations about its most probable value. In either case the objective cannot be achieved in any degree without imposing controls on the manufacturing process, in order to insure the systematic elimination of assignable

causes of fluctuation, thus reducing the range of dispersion of the individual values. The visible effect of such control is to produce a unimodal frequency distribution of the characteristic bell shape, which, in the ideal case of maximum control, may be expressed by the normal or Gaussian law:

$$f(X, \bar{X}, \sigma) = \Phi_o(z) = \frac{1}{2\pi} e^{-0.5z^2} \quad \ldots\ldots (9)$$

in which $e$ is the base of Napierian logarithms and

$$z = \frac{X - \bar{X}}{\sigma} \quad \ldots\ldots (10)$$

The distribution functions of real properties generally deviate more or less from the ideal form expressed by Eq. 9, even if the degree of production control imposed is high. They are mostly asymmetric and the most probable or peak value of the distribution—its modal value $X_o$—differs from the mean value $\bar{X}$ (see Fig. 2). The first two terms of the Gram-Charlier series[6] containing the skewness $k$:

$$f(X, \bar{X}, \sigma, k) = \Phi_o(z) - \frac{k}{6}\Phi'''_o(z) = \Phi_o(z)\left[1 - \frac{k}{6}(3z - z^3)\right] \quad \ldots\ldots (11)$$

(in which $\Phi_o(z)$ is given by Eq. 10 and $\Phi'''_o(z)$ denotes its third derivative) will be found to furnish fair approximations of real frequency distributions of controlled physical properties and phenomena (Fig. 3).

Eqs. 10 and 11 can also be used to advantage if the probability of occurrence is based, not on experience expressed by series of direct observations, but on the knowledge or assumption of an a priori probability $P_p$, since they represent good approximations of the binomial law $(P_p + P_q)^n$. This law is unsuitable for practical computation if the result desired is the probability that an individual occurrence will exceed, rather than equal, $P_p + j\sigma$. If $P_p$ is known, the standard deviation $\sigma$ and the skewness $k$ are given by the expressions

$$\sigma = \sqrt{P_p P_q n} \quad \ldots\ldots (12a)$$

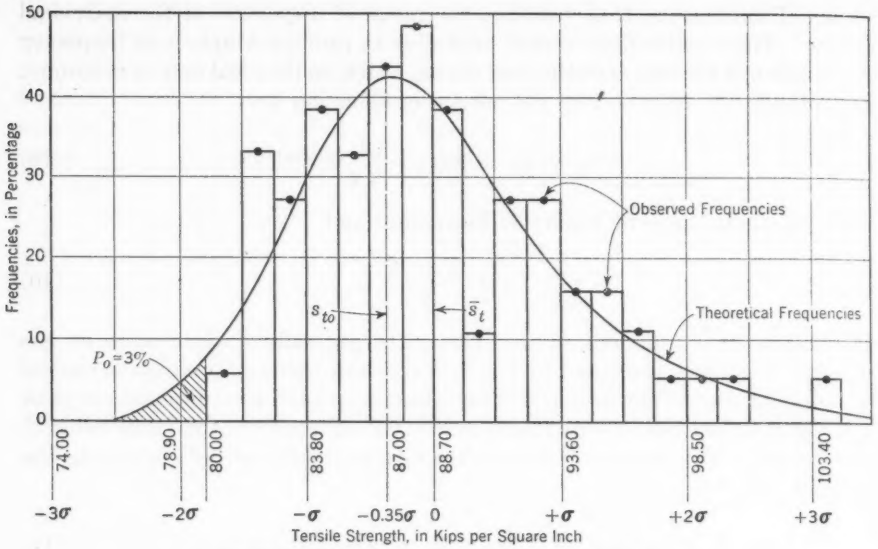
and

$$k = \frac{1 - 2P_p}{\sigma} \quad \ldots\ldots (12b)$$

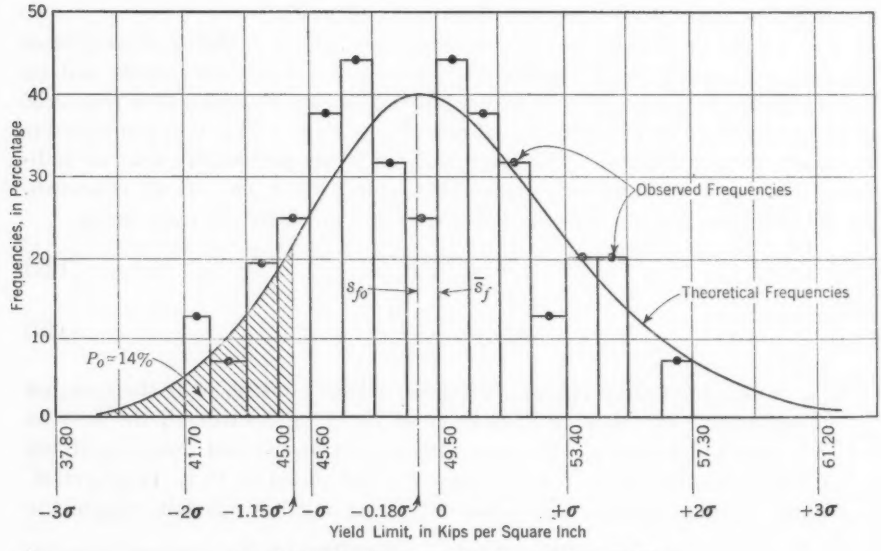
If a physical property is manufactured without control, or if the designer has no conclusive evidence or knowledge of its being controlled, the best he can do toward determining the probability that individual values will fall within specified intervals is to use the theorem[7] proposed by P. L. Tchebycheff. According to this theorem, the probability that a value will fall outside the limits $\bar{X} \pm j\sigma$ is always smaller than $\frac{1}{j^2}$, regardless of the shape of the distribution function. The effect of quality control on the safety of structures is evident from a comparison of the accuracy of predictions based on the

[6] "Probability and Its Engineering Uses," by Thornton C. Fry, D. Van Nostrand Co., Inc., New York, N. Y., 1928, p. 261.

[7] "A Treatise on Probability," by J. M. Keynes, Macmillan Co., New York, N. Y., 1929, p. 355.




(*a*) TENSILE STRENGTH: $\bar{s}_t$ = 88.70 Kips per Square Inch; $\sigma$ = ±4.9 Kips per Square Inch; and, $k$ = +0.72




(*b*) ELASTIC LIMIT: $\bar{s}_f$ = 49.5 Kips per Square Inch; $\sigma$ = ±3.9 Kips per Square Inch; and, $k$ = +0.27

FIG. 3.—COMPARISON OF OBSERVED AND THEORETICAL FREQUENCY DISTRIBUTIONS

Tchebycheff theorem with those derivable, under controlled conditions, by Eq. 10 or 11.

When imposing a standard of control on the manufacturing process of a given physical object the question necessarily arises as to whether the value aimed at should be stipulated as minimum limit or whether the inevitable dispersion of values should be controlled about an expected average. It can easily be shown that the setting of minimum limits, although still the prevalent conventional procedure, is considerably less effective in safeguarding such a limit and less reliable than the control of the fluctuations about an expected mean or most probable value. The current opinion that adequate safety can be insured only by specifying minimum limits is unfounded.

As an illustration consider the published results[8] of acceptance tests of structural shapes of silicon steel for the towers of the Golden Gate Bridge in California. The statistics have been computed, and frequency distributions of tensile strength and yield point developed from the published data, and the latter have been expressed in the form of Eq. 11 (see Fig. 3). The specifications for this bridge required minimum values of $X_m = 80{,}000$ lb per sq in. and 45,000 lb per sq in. for the tensile strength and the yield point, respectively. The integral

$$\int_{-\infty}^{X_m} f(X, \bar{X}, \sigma, k)\, dX = p \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

represents the "defective fraction" or the probability that an individual value will fall below the minimum limit stipulated. If two samples are taken from each melt and if the melt is accepted only when both samples prove satisfactory,[9] the probability that a given melt will be rejected will be $p^2 + 2\,p\,(1 - p) = 2\,p - p^2$ according to the binomial law. Hence, it will be more probable that a melt will be rejected than that it will be accepted only if $(2\,p - p^2) > 0.5$ or if $p > 0.3$; that is, if the defective fraction exceeds 30%, a possibility that is conceivable but under conditions of most inadequate control. Under the stipulation of a minimum limit, therefore, the chance of eliminating defective melts by the usual procedure of sampling is real only if the manufacture of the product is subjected to a practically inconceivable degree of control. Even under adequate conditions of control the occurrence of values lower than the stipulated minimum is still possible; this probability is computable by a statistical interpretation of the tests. The only possibility of reducing this probability practically to zero is by controlling the dispersion of values about an expected mean value, thus defining the minimum limit in terms of the probable extreme range of fluctuations below this mean value. The distribution functions reproducing the results of the check tests determine the range of fluctuation $\pm\, j\,\sigma$ about the mean, or the most probable value of $X$, in such a manner as to enable the designer to make the probability of an individual value smaller than $\bar{X} - j\,\sigma$ lower than any arbitrarily chosen figure. For a reasonably high level of control the introduction of $j = 3.5$ with regard to fluctuations about

[8] "Structural Applications of Steel and Light-Weight Alloys," A Symposium, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 1328–1329.

[9] Standard Specification for Structural Steel, A.S.T.M., A-7-42, Philadelphia, Pa., 1943.

the mean value usually reduces this probability to less than 0.0001, as can be shown by evaluating the integral of Eq. 9. Fig. 3 shows that by controlling the dispersion of values around the mean, or the mode, and introducing the probable maximum range of fluctuations $\bar{X} - 2.5\,\sigma$ or $X_o - 2.15\,\sigma$ for tensile strength and $\bar{X} - 2.75\,\sigma$ or $X_o - 2.57\,\sigma$ for elastic limit, the minimum limit is safeguarded more effectively than by stipulating it in the conventional manner. This procedure should be adopted in engineering practice, and experience should be represented by the mean value $\bar{X}$ of the observations, or test results, together with its standard deviation $\sigma$ and its skewness $k$. Only in cases in which the available data are unsuitable for statistical interpretation owing to deficiencies in scope and character will it be necessary to stipulate extreme limits for all possible values of the phenomenon or the characteristic considered. Only then will it be necessary to limit the maximum range of fluctuation for assumptions based on collateral information, and to introduce the midpoint between the extremes as a tentative approximation of the most probable value.

According to Eq. 3, the safety factor is derived from the ranges of probable maximum deviation of strain and resistance from their expected (that is, their most probable) values, which are to be considered the actual design values. Since the real distribution functions will usually be asymmetric and since, therefore, the mean value will not be identical with the modal value, it will be necessary to determine the modal value of $X_o$ as the maximum of the function $f(X, \bar{X}, \sigma, k)$ by solving the equation

$$\frac{d}{dX}[f(X, \bar{X}, \sigma, k)] = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

and introducing it as the design value and as the reference value for determining the relevant range of fluctuation.

## Classification of Influence Bearing upon the Safety of Structures

The factor of safety is affected by two groups of influences:

(1) Influences that control the strain induced in the structure or the load that produces such strain; and

(2) Influences that control the resistance of the structure or its carrying capacity.

In both groups the existence of assignable causes as well as of chance causes may be presumed. Conditions for controlling all relevant properties and phenomena leading to the complete elimination of assignable causes and to the reduction of chance causes to a constant system of equivalent effects, which have been successfully imposed on highly organized industrial mass production, will necessarily be less perfect in the more individual, less standardized organization, for the production of such units as civil engineering structures. In the latter case, economic considerations prevent the rigorous enforcement of conditions of maximum control, and they define a certain state of control as a limit beyond which assignable causes are to be considered

chance causes. Beyond such a specified limit any further elimination of assignable causes would not be economically justified, since the control measures, when applied beyond a reasonable scope, are not only expensive, but may become a serious obstacle, impeding the rate of progress of construction so considerably as to outweigh all economic advantages. It is reasonable, therefore, not to lay too much stress on the discrimination between assignable and chance causes, but to regard the difference as a difference of grade rather than of principle, and thus to extend the concept of "chance causes" to cover also assignable causes left unidentified, completely or beyond a certain limit, because their identification would not be worth while.

For some purposes it will even be convenient arbitrarily to present genuine assignable causes as chance causes, particularly in connection with problems of design. Extensive and intricate computations, such as are sometimes required for the exact design of complex but relatively unimportant structures, are disproportionately accurate from an economic point of view. For the most part, the maximum range that embraces all possible values of the several characteristics of design, however, can be evaluated by a short-cut method; or such a range is known by experience. By introducing the midpoint value between the extremes as the "design value," a range of artificial "chance fluctuations" is created, arbitrarily, which will affect the safety factor by expressing it as a function of the accuracy of the design. In a highly refined design, in which the conditions of strain and resistance are reproduced perfectly, genuine chance fluctuations, mostly controlled, will affect the safety factor within comparatively narrow limits. A crude design, requiring only a rapid computation of approximate values (and therefore marked by a high degree of uncertainty represented by a wide range of "artificial chance fluctuations") will lead to a considerable increase in the safety factor required.

## Superposition of Individual Influences

In order to superimpose the fluctuations of the individual constituent qualities of strain and resistance, respectively, the method of statistical superposition must be introduced. In the functional relationship

$$X = F(x_1, x_2, x_3, \cdots, x_n) \quad \ldots\ldots\ldots\ldots\ldots\ldots (15a)$$

the characteristics $x_1, x_2, x_3, \cdots, x_n$ are more or less controlled around the corresponding "design values" $x_{1o}, x_{2o}, x_{3o}, \cdots, x_{no}$ (which may be the means or the modal values) and the value of the principal characteristic $X$ is subject to fluctuations $\pm j\,\sigma$ about its "design value":

$$X_o = F(x_{1o}, x_{2o}, x_{3o}, \cdots, x_{no}) \quad \ldots\ldots\ldots\ldots\ldots\ldots (15b)$$

The range of fluctuations $\pm j\,\sigma$ can be determined by applying the following rule for the evaluation of the standard deviation of the resultant characteristic from the standard deviation of the constituents:[10]

$$\sigma = \sqrt{a^2_1\,\sigma^2_1 + a^2_2\,\sigma^2_2 + a^2_3\,\sigma^2_3 + \cdots a^2_n\,\sigma^2_n} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

[10] "Economic Control of Quality of Manufactured Product," by W. A. Shewart, D. Van Nostrand Co., Inc., New York, N. Y., 1931, p. 393.

in which

$$a_n = \left(\frac{\partial F}{\partial X_n}\right)_{x_n=\bar{x}_n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Eq. 16 shows that owing to the relatively slight probability that all the component fluctuations will occur simultaneously, and in full intensity, the fluctuations of the resultant characteristic will be included within considerably narrower limits than might be inferred from the algebraic addition of the component fluctuations.

For the large number of material properties that have a bearing on the design, the level of control maintained will scarcely be uniform. The conditions of production vary considerably, not only with regard to the different structural materials but with regard to the individual stages of the construction process as well. Conditions of maximum control of certain properties may exist simultaneously with moderate control of other phenomena. In order to superimpose the fluctuations of the individual constituents of strain and resistance in spite of the different shapes of their distribution curves, the factor $j$ (which relates the probable maximum range of fluctuations with the standard deviation of the frequency distribution) must be chosen individually for each component quality with regard to the particular degree of production control maintained. This must be done in such a way as to insure that the probability of occurrence of values beyond the range of variation considered shall be equal for each and every constituent. In other words, for strain all integrals $\int_{X_o+j\sigma}^{+\infty} f(X, \bar{X}, \sigma, k)\, dX$ of the distribution functions of the component qualities must be equal; and, for resistance, all integrals $\int_{-\infty}^{X_o-j\sigma} f(X, \bar{X}, \sigma, k)\, dX$ must be equal.

Assuming that different levels of control exist in different parts or in different stages of a construction process, distinguished by the factors $j_k$, $j_l$, $j_m$, etc., and leading to equal probabilities of defective individual values of the respective qualities, and considering absolute ranges of variability $\Delta_n$ of certain properties or phenomena, the range of fluctuation of the resultant characteristic $X$ may be written in the form

$$\Delta X = \sqrt{j^2_k \Sigma(a_k \sigma_k)^2 + j^2_l \Sigma(a_l \sigma_l)^2 + j^2_m \Sigma(a_m \sigma_m)^2 \cdots + \Sigma(\Delta_n)^2}. \quad (18)$$

## Safety Factor and Probability of Failure

The safety factor, as expressed by Eqs. 2 and 5, is derived from the condition that it should prevent failure even if a highly unfavorable and very improbable upward fluctuation of strain or load were to coincide with a particularly unfavorable and improbable downward fluctuation of resistance or carrying capacity. If $P_s$ denotes the probability of occurrence of an extreme strain $(s_{ao} + \Delta s_a)$ and $P_r$ is the probability of occurrence of the minimum resistance $(s_{ro} - \Delta s_r)$, the probability of coincidence of both extremes—that is, the probability of failure $P_f$—will be, according to statistical principles

$$P_f = P_s P_r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Denoting by $n$ the expected number of load repetitions during the presumed period of service of the structure, $n\,P_f$ represents the number of load repetitions during this period which might actually lead to failure. In order to prevent even a single occurrence of this critical state of the structure it is necessary to stipulate that $n\,P_f < 1$, or that $P_f = P_r\,P_s < \frac{1}{n}$.

Since, for known distribution functions of strain and resistance the choice of $P_s$ and $P_r$ determines the respective ranges of fluctuation, the foregoing

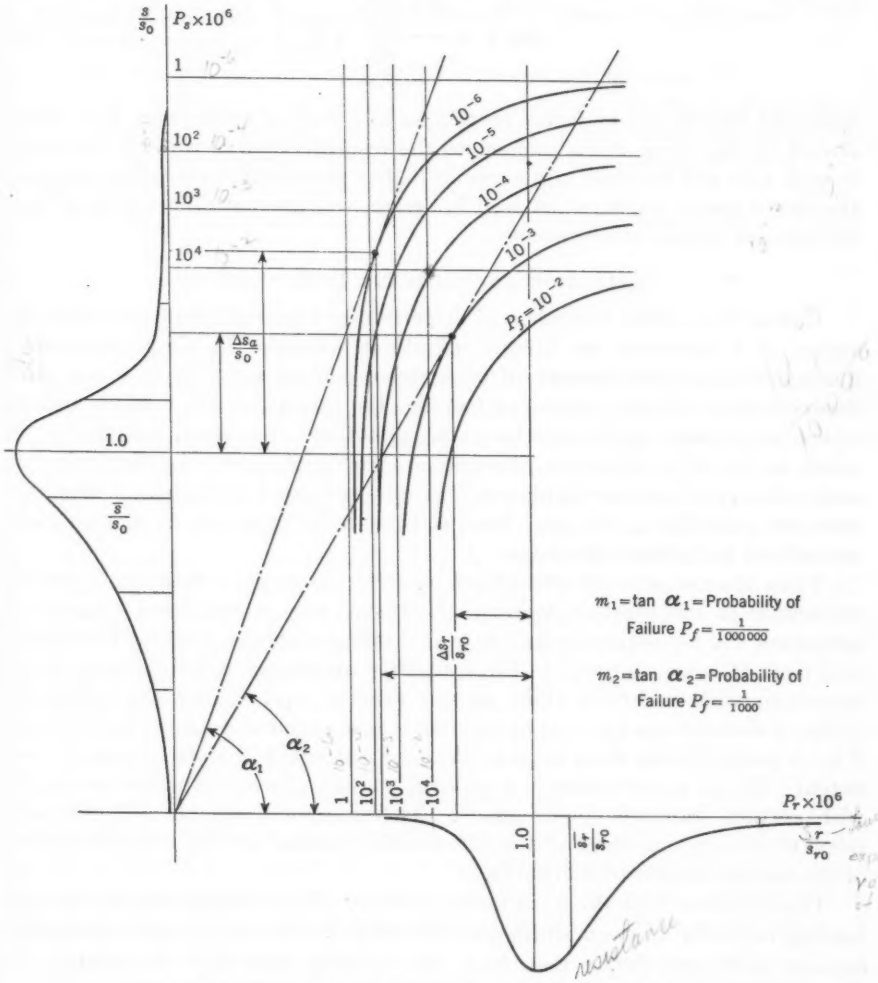



FIG. 4.—CURVES OF EQUAL PROBABILITY OF FAILURE (SEE EQ. 21)

condition expresses the necessary relation between these ranges for every probability associated with the prevention of failure. Drawing the distribution function of the resistance as abscissas and that of strain as ordinates, or

indicating on each the specific range of fluctuations $\frac{j\,\sigma_s}{s_{ao}}$ and $\frac{j\,\sigma_r}{s_{ro}}$, respectively, the pertinent probability of values falling outside this range, it is possible to trace the family of curves of equal probability of failure (see Fig. 4)

$$P_f = P_s\,P_r = \text{constant} \qquad (20)$$

The safety factor for each point on those curves is represented by the slope of the line connecting the respective point with the origin because

$$\tan\alpha = \frac{1 + \frac{\Delta s_a}{s_{ao}}}{1 - \frac{\Delta s_r}{s_{ro}}} = \zeta \qquad (21)$$

Since the highest safety factor pertaining to a certain probability $P_f$ is represented by the slope of the tangent to the respective curve through the origin (which does not intersect any curve of higher probability) this slope expresses the safety factor which, when used in design, will prevent failure with at least the selected degree of probability.

## Selection of Relevant Influences

Causes that affect the range of dispersion of the strain and the resistance values of a structure are almost infinite in number. A single component quality, such as the strength of concrete in compression, for instance, itself depends upon a large number of factors—the quality of the cement, quality of the aggregates, quality of the water, quantities of cement, aggregate, and water in the mix, influences of workmanship (mixing, transporting, placing, and curing), and uncontrollable external influences, such as changes of temperature and humidity of the air. Most of these individual factors, in their turn, are subject to further influences.

Thus, the compressive strength of concrete is actually a function of several thousands of independent factors. However, only a restricted number of influences will be assignable and relevant during a certain stage in the transition from the raw material to the completed structure. The designer, being concerned with a certain stage of this process, may regard the assignable causes of fluctuations relevant to previous stages as genuine chance fluctuations, if he is satisfied that those stages have been subject to a state of control consistent with the manufacture of a uniform product which is expected to comply with certain standard requirements. Therefore, the number of influences relevant in a certain stage of the construction process is only a small fraction of the actual number of influences.

The influences that affect the factor of safety will be divided into two groups bearing respectively upon strain and resistance or (whenever such correlation appears more expedient) upon load and carrying capacity. In addition to these principal groups, an intermediate group, embodying influences of the method and procedure of computation of strain, must be introduced.

The fundamental types of influences are enumerated in the following outline, separated into classes and groups in conformity with the aforementioned divisions.

Group A. Causes of Fluctuation in Strain.
- I. Uncertainty and variability of loading conditions.
  - a. Dead load.
  - b. Live load (including dynamic effects).
- II. Uncertainty and variability of external conditions that are independent of the load.
  - a. Change of temperature.
  - b. Wind forces.
  - c. Uncertainty of behavior of the subsoil.

Intermediate Group; Causes of Uncertainty of Strain Computations.
- III. Variation of rigidity.
- IV. Imperfection of methods and shortcomings of assumptions.
  - a. Accuracy of method and tolerances of numerical computation.
  - b. Inadequacy of assumptions concerning initial and boundary conditions, stress concentration, and secondary strain.

Group B. Causes of Fluctuation of Resistance.
- V. Uncertainty and inaccuracy of the assumed mechanism of resistance.
  - a. Inaccuracy or inadequacy of conceived mechanism.
  - b. Variability of resistance limits of materials.
- VI. Variation of structural dimensions.

In order to illustrate the determination of the appropriate design values of the individual component qualities and characteristics, and the establishment of ranges of dispersion about these values, some of the typical influences enumerated herein will be analyzed. Since neither the scope nor the character of the data that are assumed to represent past experience concerning these influences is adequate to the purpose of the present investigation, the following examination of the individual causes is tentative and suggestive rather than conclusive. Its main object is the indication of methods and of procedure to be adopted, deficiencies in the data to be supplied by future research. Consequently, the numerical values presented will have to be considered with some reservation, the main emphasis being laid upon the rational insight obtained into the true conditions of safety in engineering structures.

## PART 2.—ANALYSIS OF PARTICULAR INFLUENCES

### Dead Load

The fluctuations of the dead-load values are caused by the variability of the specific weight of the materials concerned and of the dimensions of the component parts of the structure. Therefore, the combined consideration of the tolerances of weight and dimensions will lead to a reasonably accurate estimate of the variation of the dead load.

Moreover, a discrepancy may generally be assumed to exist between the design load (mode of an assumed distribution curve) and the mean value due partly to the inaccuracy of the weights estimated (which are more frequently underrated than overrated), and partly to slight alterations of design during construction; also (except in very simple structures) the actual distribution of

the dead load is usually so intricate that its design value represents at best an approximation of the actual condition. It is a frequent experience that computations made for control purposes, after the structure has been completed, yield dead loads considerably in excess of the design load initially estimated. The following formula for the dead load $w_d$ is suggested:

$$w_d = 1.10\,\bar{w}_d\,(1.0 \pm 0.10) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

in which $\bar{w}_d$ denotes the average dead load determined on the basis of the conventional values of the specific weights and of the nominal values of the dimensions.

## Live Load

*General.*—Among the many instances of inconsistency between the accuracy and the refinement of design methods and the inadequacy and vagueness of the underlying assumptions, specifications concerning live loads are most conspicuous. The specification of service conditions is one of the unsatisfactory features of the conventional design. The values recommended are arbitrary and, for the most part, have no structural significance, emanating often from non-engineering quarters. A clear-cut distinction between everyday conditions of service and fictitious conditions is noticeably lacking. It is imperative that the service conditions be identified as reliably as the strain or resistance of the structure. It must be admitted, however, that the difficulties of defining the service load on which a judicious and rational design can be based are impressive, owing to the considerable spatial and temporal fluctuations of the (static) service weights, as well as to the variability of the accompanying dynamic effects.

The design live load should: (*a*) Represent the most probable actual service conditions of reasonably high frequency of occurrence; and (*b*) provide for such increase of the service load as may be expected, reasonably, during the assumed period of service of the structure, considering the general trend as well as local circumstances. Allowance should be made for possible, yet comparatively infrequent, adverse conditions of service, in the form of appropriate ranges of fluctuations about the design value, and this allowance should be embodied in the factor of safety.

In general, conventional specifications do not fulfil any of the foregoing requirements; the design loads stipulated represent, for the most part, highly unfavorable conditions, the occurrence of which is not only infrequent but improbable. Safety factors currently adopted are neither concerned with, nor do they refer to, the probability that such design live loads may actually occur, so that the numerical safety attributed to the design has no relation to the actual safety of the structure.

*Intensity and Distribution of Live Loads for Buildings.*—Live-load specifications should distinguish between two groups of buildings with regard to the purpose they are expected to serve; namely, buildings for human occupancy, and buildings for industrial purposes and storage. The principal difference between these groups is the character of the service loads. In buildings for

human occupancy, the intensity of dead loads and live loads may fluctuate widely, whereas in industrial, and especially in storage buildings, there is an almost constant maximum load. Since the simultaneous occurrence of such load on all parts of the structure represents actual service conditions, the probability that other conditions may occur is irrelevant.

*Intensity and Distribution of Live Loads for Bridges.*—The difference between the design live load of highway bridges and of railroad bridges resembles somewhat the difference between buildings for human occupancy and buildings for industrial use. A highly variable load intensity on highway bridges necessarily leads to design loads dependent upon the area loaded and subject to a wide range of chance fluctuations, whereas the service conditions of railroad bridges require the introduction of definite maximum design loads.

Adequate design loads for highway bridges can be determined, if information is available as to the character and dimensions of the most important traffic units, their weight and weight distribution, the distribution of different units along the traffic lane, velocity of traffic and dynamic effects, and the trend of future development. The only evidence so far collected that is useful practically has been for purposes of traffic research. A certain amount of information can be gleaned from these data, suggesting the following conclusions:

(1) The standard load of the modern highway bridge is the motor vehicle, of which at least three different kinds (units) must be considered—the truck, the rapid transit bus, and the passenger car. Widely varying types of such units are to be found on the highway and the average weight of such units remains considerably less than that of its heaviest type.

(2) The heaviest two-axle truck used to any considerable extent at present weighs, fully loaded, not more than 50,000 lb. It is about 30 ft long and thus represents an equivalent load of some 1,700 lb per lin ft of traffic lane. Since two thirds of the weight is carried by the rear axle the maximum concentrated load to be considered per lane is roughly 34,000 lb. The equivalent load intensity of the rapid transit bus approaches that of the heaviest truck but the distribution of axle loads is more favorable. On the other hand, the heaviest passenger car, about 20 ft long, weighs, fully loaded, not more than some 10,000 lb, resulting in a load intensity of about 500 lb per lin ft of lane. According to registration records, between one third and one fifth of all motor vehicles are trucks. Although no data are available concerning rapid transit buses, their number may be assumed to be comparatively small. It is anticipated that future expansion in the number of motor vehicles will be in passenger cars rather than in trucks. A reduction in the ratio of trucks to the total number of vehicles registered, amounting to one sixth by 1960, has been predicted.[11]

(3) The character and distribution of traffic vary with local conditions; but such conditions may change rapidly with the development or movement of industry. Since prediction concerning such matters will be highly unreliable, it is reasonable to avoid introducing too sharp regional differentiations in

[11] "Motor Transportation—A Forward View," A Symposium, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 1574.

specifications for traffic loads. A certain degree of differentiation is justified, however, with regard to the volume of traffic, which will affect the number of load repetitions, but not the load intensity.

(4) One factor that exerts considerable influence upon the load intensity is the distance between motor vehicles moving along the same traffic lane. Evaluating investigations reported by the U. S. Bureau of Public Roads,[12] the relation between the center to center spacing $l$ of vehicles and their traveling velocity $v$ can be approximated by the formula

$$l = \frac{L}{1 - 0.15\, v/V}\,(1 \pm 0.20) \qquad (23)$$

valid for $0 \leqq v \leqq 60$ miles per hr, in which $V = 10$ miles per hr. For trucks, $L = 40$ ft; for passenger cars, $L = 30$ ft. A speed of 10 miles per hr denotes extremely congested traffic with all vehicles moving at equal speed; speeds exceeding 40 miles per hr are possible only if the rate of movement of each vehicle is independent, distances between adjacent vehicles being such that the equivalent loads exerted are very small.

In formulating specification for the design loads of railroad bridges, the frequency of occurrence of different types of load units is irrelevant. The design load should represent the heaviest train expected to run on the given track within the assumed period of service.

*Dynamic Effects of Live Loads.*—The action of the service load is generally considered as static in the design of buildings and as dynamic in the design of bridges. Actually, however, only a limited number of structures is subject to genuinely static load action (dams, aqueducts, etc.). The loads acting on most structures are variable and transitory. Whether the effect of such loads is static or dynamic is thus a question of the rapidity of application of the strain.

The dynamic behavior of the load intensifies the strain induced in the structure and thus affects its safety. The "dynamic increment" $I$, indicating the excess of the dynamic over the static load, is conventionally considered the dynamic characteristic of the design. This concept, however expedient it may be, hardly conveys a picture of the real conditions in the structure under dynamic load action. The complex process of transfer of potential (strain) energy into kinetic (vibration) energy cannot be expressed adequately by such a simple device, based on the unjustified assumption of proportionality between the effect of a static load and that of a load moving at a certain speed. This fact, although generally recognized,[13] so far has had little effect on experimental research, which has mainly been directed at the determination of definite numerical values of the dynamic increment. It may be anticipated, therefore, that the results of such research will appear widely scattered and that final conclusions of general validity cannot be formulated.

The main reason for the inconclusiveness of impact research is the fact that only one part of the strain in a structure subject to traveling loads is functionally dependent on major assignable causes such as masses, weights,

[12] "Preliminary Results of Highway Capacity Studies," by O. K. Normann, *Public Roads*, February, 1939, Fig. 5, p. 227.

[13] "European Developments in the Study of Impact and Fatigue," by F. H. Frankland, *Civil Engineering*, April, 1940, p. 208.

velocities, dimensions, and elastic constants. The other part depends on more or less accidental circumstances and effects, such as the damping capacity of the structure, its state, the conditions of track or roadway, the wear of the wheels of the vehicles, etc. If, for the lack of a more expedient dynamic characteristic, the dynamic increment $I$ is to be retained, and only the underlying concept is to be modified so far as to make it consistent with experience, it will be necessary to distinguish between assignable influences on this increment and chance influences, these being understood to cover genuine chance causes as well as unidentified assignable causes.

The functional relation between the average value of the increment and the load impulse has been found to be very nearly linear.[14] The range of dispersion about this "line of best fit" must be expected to be greatest for structures of medium span for which vibrations represent a substantial source of strain. In such structures the range of magnitude of the frequency of natural vibrations compares with the frequency of the real load impulses; the intensity and number of such impulses are sufficient to overcome the inertia of the structure within the period that the load is acting. For these reasons, conditions approaching resonance may be reproduced. Short-span structures protected by their rigidity and the resultant high frequencies of natural vibration, and long-span structures protected by their great masses from being excited to considerable vibrations by the short action of comparatively light service loads, may be expected to show a relatively narrow range of dispersion of values of the dynamic increment. The range of fluctuation about the mean value $\bar{I}$ thus tends to decrease with spans, increasing beyond, as well as dropping below, the critical medium-span range. Practically, however, only the tendency to decrease with increasing span is manifest. For short spans the growing influence of local irregularities of the load action, and of the path of the load upon the intensity of individual impact impulses, is strong enough to reverse the opposite trend.

The genuine chance effects are of a considerably narrower range of variation in railroad bridges than they are in highway bridges. In the latter, the intensity, as well as the frequency and number of consecutive load impulses, varies within wide limits owing to the complexity of traffic conditions and the fact that the local condition of the roadway may become deteriorated to an extent that would be practically unimaginable on a railroad track in service. A similar explanation accounts for the wear of rolling surfaces of vehicle wheels —another factor that intensifies the load impact.

The mean or "line of best fit" of observations[15] pertaining to a single motor vehicle traveling at the speed $v$ may be represented by:

$$I_o = \frac{1.50}{1 + \frac{V}{3v}} \qquad (24)$$

in which $V = 10$ miles per hr. The increment pertaining to $n$ consecutive

[14] "Vibration Problems in Engineering," by S. Timoshenko, 2d Ed., D. Van Nostrand Co., Inc., New York, N. Y., 1937, p. 363.

[15] "Impact in Highway Bridges," by A. H. Fuller, *Final Report*, 2d Cong. for Bridge and Structural Eng., Springer, Berlin, 1929, p. 56.

vehicles may be derived by assuming that the probability of the simultaneous occurrence of the dynamic increment $I$ and $n$ vehicles is governed by chance. Hence the most probable increment in an observation of $n$ vehicles, $I_{no}$, is obtained by applying Eq. 16 to Eq. 24, thus

$$I_{no} = \frac{1}{\sqrt{n}} I_o = \frac{1.50}{(1 + V/3\,v)\,\sqrt{n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

The influences of the structure, and of the irregularities of the roadway and the tires, are represented by the range of fluctuations about the foregoing average. For single vehicles acting on stringers of very short span, the range may be as great as $\pm$ 100% of the average if the possibility of trucks with hard-worn tires traveling on rather worn roadways is to be provided for. The dynamic increment for $n$ vehicles of weight $W$ may be written in the form

$$w_l I_n = \frac{1.5\,w_l}{(1 + V/3\,v)\,\sqrt{n}} \left(1 \pm \frac{1.00}{\sqrt{n}}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

in which $w_l = \dfrac{\Sigma W}{n\,l}$ denotes the load intensity in pounds per linear foot of traffic lane. If the structure is long enough to accommodate more than a single load unit per lane, the load intensity $w_l$ becomes a function of the traveling speed of the load, because the spacing between the consecutive load units increases with increasing speed. A critical speed for which the product of load intensity and dynamic effect $(1 + I)\,w_l$ attains a maximum value, and which represents the traveling speed of the design load, is obtained by a differentiation of this product. With the resulting $v/V = 1.15$, the "critical" velocity of the design load is shown to be not more than 11.5 miles per hr. Only for comparatively short spans does one single load unit per lane, traveling at maximum speed, represent the design load.

Any presentation of the dynamic effect of the load is incomplete without a knowledge of the velocity of strain, the maximum number of consecutive load impulses, and the total number of impulses on the structure during its entire expected period of service.

In short-span steel bridges for railroads, for which the share of the dead-load strain in the total strain is almost negligible, the velocity of specific impact strain may attain a magnitude of 0.8% per sec for mild steel, the assumption being that the ultimate strain may almost attain the yield-point limit if the service load is acting under the most unfavorable conditions. With increasing spans the velocity of strain due to vibrations becomes dominant, which is in the range of magnitude of 0.2% to 0.3% per sec for mild steel. In large-span structures the maximum rate of specific strain, due to transitory load movement, will generally remain below 0.05% per sec. The rates of strain application in highway bridges are considerably lower than those in railroad bridges because the "critical" speed of the design load is less than one fifth of the traveling speed of fast trains. The strain rates in structures of low-alloy steel are necessarily higher than those for mild steel; and the values for reinforced concrete structures are considerably lower owing to the high dead-load ratio.

Load repetitions on single-track railroad bridges seldom attain 1,000 per day even for very dense traffic of about 80 to 100 trains daily, since the critical impulses in short-span structures are exerted by the locomotive driving axles only. The number of repetitions of the load of railroad bridges may thus exceed ten million for short-span structural parts if a 30-yr period of service is assumed. This number decreases rapidly with increasing spans.

In order to estimate the number of load repetitions as well as the number of consecutive load impulses on highway bridges a certain amount of evidence concerning the volume and distribution of traffic is necessary. The evaluation of traffic records[16] suggests that dense traffic is concentrated over eight consecutive hours of the day only, the maximum hourly volume reaching 6% of the daily total. For short-span structures and structural parts, the number of repetitions of the design load is determined by the number of trucks passing. This number during a 30-yr period of service will probably reach ten million on main city thoroughfares with 4,500 vehicles per day per lane. For medium and large spans, however, the design load consists of a congested, slowly moving row of vehicles extending over the entire span or the critical length of the influence line. Its maximum frequency (although extremely improbable) would be reached if all the traffic during this period were distributed exclusively in groups representing the design load. The total number of design-load repetitions during the expected period of service would thus decrease rapidly with increasing spans. The numbers estimated for single lanes are considerably less than those for multiple-lane bridges owing to the small probability that full design loads will occur at one time on different lanes.

### Changes of Temperature

Temperature changes in a structure involve chance fluctuations. The probability that extreme temperatures will occur is comparatively small. Changes of temperature within a restricted range, however, will occur frequently enough to justify their inclusion in any study concerning service conditions. Therefore, temperature changes should be divided, according to frequency of occurrence, into changes affecting normal conditions of service and changes over an extreme and infrequent range affecting the safety factor.

### Wind Forces

The magnitude and distribution of wind forces are chance events. Within a certain range of wind pressures the frequency is comparatively high and, consequently, the structure can be expected to withstand wind pressure permanently and with a degree of safety equal to that required for the principal service loads. Extreme values of wind forces, such as those that occur during severe storms, need be expected on rare occasions only. Hence, wind forces should be introduced into the actual design only as far as moderate and frequently occurring values are concerned, whereas maximum effects should be anticipated in the factor of safety. Long-time records of wind velocities will furnish the material for the preparation of distribution curves from which design values and fluctuations can be derived.

[16] "An Automatic Recorder for Counting Highway Traffic," by R. E. Craig, *Public Roads*, May, 1938, p. 48.

### Uncertainty in the Behavior of the Subsoil

The foundation is an integral part of the structure, and its behavior, mainly depending upon the behavior of the subsoil, exerts considerable influence on the state of strain in statically indeterminate structures. The strain resulting from such movement in the structure is either entirely neglected in the conventional design (being based mostly on the assumption that the supports always remain fixed or that they settle uniformly, by equal amounts) or it is regarded as of secondary importance. The movement of the subsoil being a reality, this attitude is not justified.

Probably maximum values of settlement differences should be estimated with the aid of selected methods defined by soil mechanics, and the range of fluctuation of the dead-load strain, such as might actually occur when these differences are most unfavorable, should be established. Such fluctuations considered as chance events, varying about the design values of the dead-load stress, will necessarily increase the dispersion of the individual stress values and, consequently, will increase the required factor of safety. Owing to the high degree of rigidity and the almost perfect elasticity of the subsoil under the action of transient loads, the mutual effect of subsoil and structure is restricted to dead-load stress only.

### Variation of Structural Rigidity

The rigidity of a structural part is expressed as the product of a sectional value, such as the moment of inertia or the area, and the respective modulus of elasticity of the material, divided by the length of the member. The degree of rigidity will vary, therefore, with changes of either the modulus of elasticity or the sectional value, or both, fluctuations in length being insignificant.

It is evident that the rigidity of all parts or members of one and the same structure will not necessarily be subject to uniform variation. Allowance must be made for the fact that the rigidity of individual parts or members will differ among themselves. The effect of such variation upon the strain of redundant structures may be determined by a method which has been called "analytical experiment" by Hardy Cross,[17] M. Am. Soc. C. E. This method consists of the actual computation of certain characteristics of strain in a structure subject to any possible combination of variation of rigidity in its individual parts or members.

In metallic structures subject to air temperatures only, no change of the initial rigidity need be expected unless the cross sections are reduced by deterioration. In concrete and reinforced concrete structures, however, the additional deformation (creep) that occurs under action of sustained compressive loads makes the initial modulus of elasticity appear to decrease. This apparent decrease affects the state of strain, and thus the safety, of plain and reinforced concrete in which sustained compression is a dominant phenomenon, such as redundant large-span arches and pre-stressed structures.[18]

---

[17] "Dependability of the Theory of Concrete Arches," by Hardy Cross, *Bulletin No. 203*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

[18] "Theory of Wide-Span Arches in Concrete and Reinforced Concrete," by A. M. Freudenthal, *Publications*, International Assn. for Bridge and Structural Eng., Vol. 4, Zurich, 1936, pp. 249–264.

### Accuracy of Methods and Tolerances of Numerical Computation

Methods of computing strain characteristics are generally based upon the assumption of perfect elasticity. This assumption is justified for metallic structures under air temperatures, and it is correct even for concrete structures strained within the range of the conventional working stresses. If the strict methods of the theory of elasticity are applied to the design, and if the respective boundary conditions are selected so as to reproduce the actual behavior, the resulting strain values are within the range of accuracy of the assumed boundary conditions.

If short-cut methods are applied as substitutes for a strict theoretical analysis, the range of error must be compensated by an appropriate increase in the general factor of safety. The functional dependence of the safety factor on the relative crudeness of the design method is thus established. The maximum range of possible error must be ascertained reliably and subsequently introduced as a constituent chance fluctuation.

### Boundary Conditions, Secondary Strain and Stress Concentration

Boundary conditions are expressed in terms of displacement and the angular deflection of end sections. The boundary conditions for the supports may be derived from observations of the movement of the foundations and the subsoil. There are cases, however, in which boundary conditions are chosen rather arbitrarily or, to simplify the mathematics, without satisfactory evidence that the choice is rational

The manner in which the boundary conditions for end sections are assumed will considerably influence the actual safety of members subject to bending or buckling. It is doubtful that any assumed degree of fixity at the ends will ever be realized precisely in the actual structure. Therefore, minimum and maximum values of the probable angular deflection at the ends should be estimated, and the mean of these values should be selected as the design value, the range of variation about the mean being considered a constituent part of the safety factor.

An important group of influences upon the safety of structures is represented by the effects of secondary strain and stress concentrations, mostly occurring at or near structural connections—thermal stresses in welds, stress concentrations around rivets, residual stresses due to rolling or cold-working, shrinkage stresses in reinforced concrete, and stress concentrations produced by surface conditions due to corrosion. The magnitude and effect of all these influences depend on a large number of chance causes and vary considerably, often in a rather erratic manner. Their effect upon the strain should be reflected in the design by reducing the resistance values rather than by increasing the intensity of the strain. In this paper, therefore, they will be treated in connection with resistance problems.

### Uncertainty and Inaccuracy of the Mechanism of Resistance

The resistance of engineering structures is affected by the character of the load, by external conditions, and by the characteristic properties of structural

materials. In order to reproduce the real phenomenon in a form suitable for design, abstractions and assumptions must be introduced.

Structural resistance is defined, generally, with reference to the state of strain which delimits the fitness for service of a structure. It is principally a function of two variables, a sectional characteristic and the resistance (strength) of the material. For practically every structural material this resistance is influenced by the rate at which strain is applied—the same material that appears permanently deformable and tough under low strain rates will appear perfectly elastic and brittle under impact strain and will show increased strength.

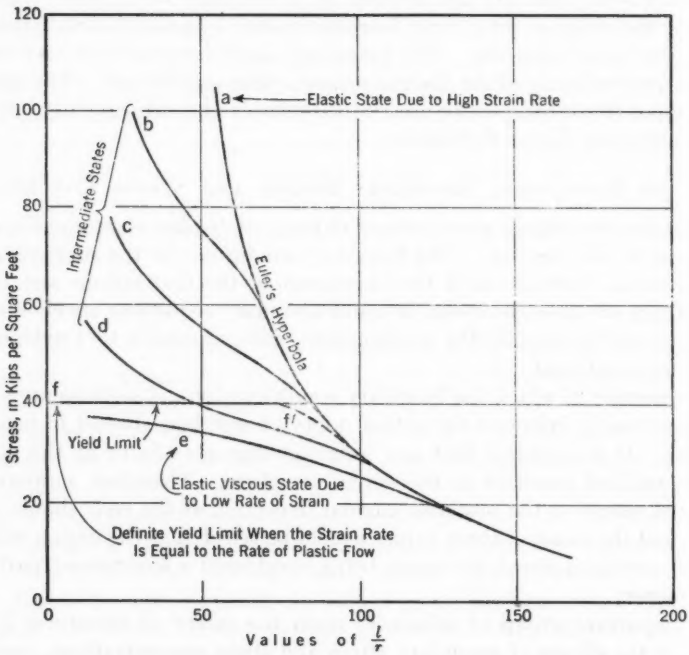



FIG. 5.—BUCKLING STRESS $s_b$ AS FUNCTION OF SLENDERNESS RATIO $l/r$ AND THE STRAIN RATE APPLIED

In certain cases resistance is not a function of sectional strength alone but also of the structure's stability, as in members subject to axial compression. The "buckling stress" embodies both sectional strength and geometrical criteria. Fig. 5, which presents the buckling stress as a function of the slenderness ratio $l/r$ and of the strain rate applied, illustrates the velocity sensitiveness of the buckling phenomenon. This aspect of the buckling problem is important in the design of compressed members under conditions of impact, under which the Euler hyperbola applies over a wider range than is conventionally considered elastic, and under conditions of creep under which the elastic range is considerably diminished.

The real resistance mechanism is normally rather complex, and it is hardly possible to conceive such a mechanism that will effectively reproduce the

actual phenomenon and which is at the same time simple enough to be suitable for practical design. Every devised mechanism, therefore, is fictitious to a certain degree; but it will embody the relevant physical qualities that are determinable by simple standardized tests. The efficiency of the mechanism is ascertained by statistically interpretable experiments, and its reliability and the range of dispersion of individual values about the average trend established. Its suitability is to be judged by the simplicity of the concept, by the closeness with which experimental results are reproduced, and by the narrowness of the range of dispersion of such results about the "theoretical" course.

The range of uncertainty characterizing a certain resistance mechanism will be affected by the state defined as undesired with regard to the safety of the structure. Satisfactory correlation of the structural resistance limits which determine this state, with the material strength limits which can be observed on reproducible test specimen, has so far been attained for the most simple conditions only.

## Uncertainty and Variation of Resistance Limits of Structural Materials

The mechanical resistance of engineering materials is affected by the strain field, the rate and duration of strain, the amplitude of strain cycles, and their number. The influence of the strain field is generally expressed by the conceived mechanism of resistance. Effects of local concentrations of strain due to notches, welds, or rivets are not embodied in the general mechanism of resistance but are considered by empirically reducing the resistance (strength)

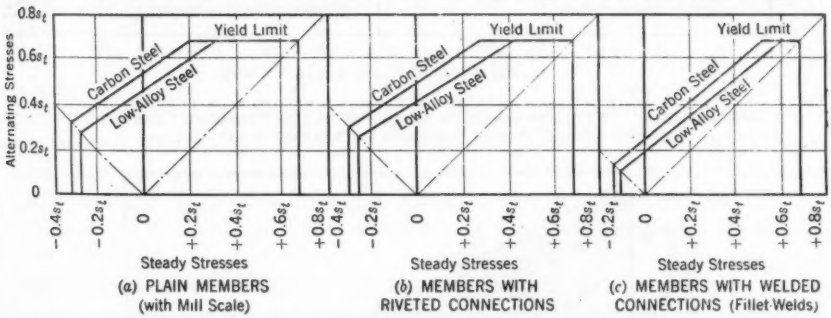



Fig. 6.—Endurance Limit in Terms of Ultimate Tensile Strength $s_t$ as Function of Amplitude of Stress Oscillations about Steady Stress

of the material in the undistorted field of strain. Since the resistance of a material relevant for purposes of structural design is its resistance at, and in, the vicinity of the heaviest concentration of strain which occurs within the areas of the structural connections, the specific resistance of the material at, or near, connections of standard shape must be determined experimentally.

The resistance-reducing effect of structural connections subject to load cycles of various amplitudes has been investigated extensively for steel.[19,20]

[19] "On the Fatigue Strength of Riveted and Welded Joints and the Design of Dynamically Stressed Structural Members Based on Conclusions Drawn from Fatigue Tests," by K. Schaechterle, *Publications*, International Assn. for Bridge and Structural Eng., Zurich, Vol. 2, 1933–1934, pp. 312–379.

[20] "On the Fatigue Strength of Riveted and Welded Joints Made of Steel St. 52," by E. H. Schulz and H. Buchholtz, *ibid.*, pp. 380–399.

Fig. 6 shows the results of the investigations in a concise form, relating the endurance limit for various amplitudes of stress cycles repeated two million times, and the tensile strength $s_t$. Owing to the different character of the connections different relationships are obtained for riveted and for welded con-

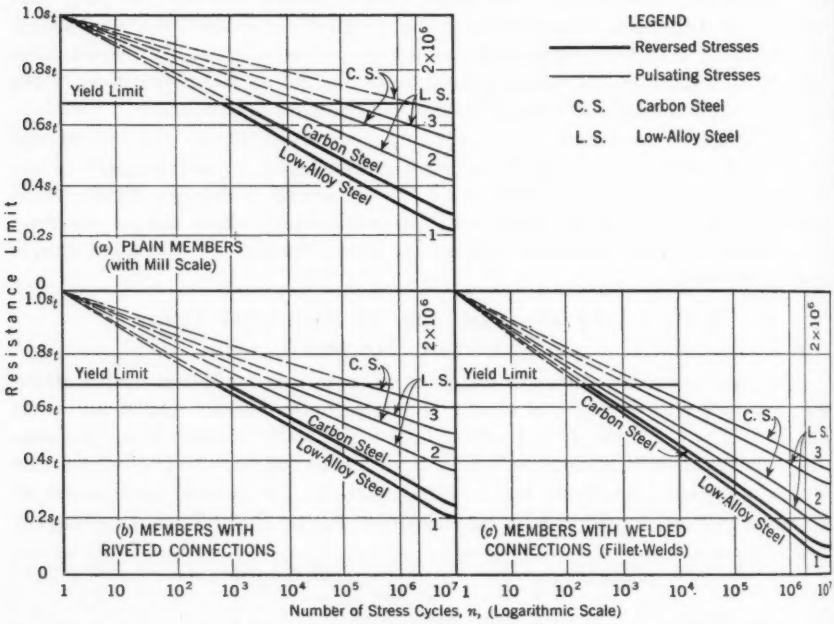



FIG. 7.—ENDURANCE LIMIT IN TERMS OF ULTIMATE TENSILE STRENGTH $s_t$ AS FUNCTION OF NUMBER OF STRESS CYCLES ("1" FULL REVERSAL OF STRESS, "2" STRESS PULSATING BETWEEN ZERO AND LIMIT, "3" STRESS PULSATING BETWEEN 0.2 $s_t$ AND LIMIT)

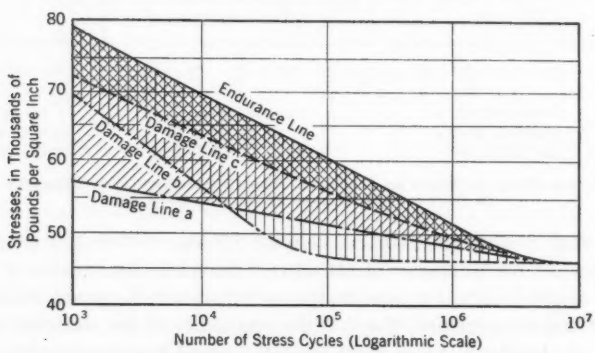

Stresses, in Thousands of Pounds per Square Inch
Endurance Line
Damage Line c
Damage Line b
Damage Line a
Number of Stress Cycles (Logarithmic Scale)

FIG. 8.—CHARACTERISTIC DAMAGE LINES AND DAMAGE RANGES

nections. Different limits are also obtained for different numbers of stress cycles (Fig. 7). The results presented in Figs. 7 and 8, however, have been derived from arbitrarily defined minimum values of the endurance limit; no statistical interpretation has been attempted and no adequate information is

available concerning the range of fluctuations of the dynamical resistance limits in general. This range will certainly exceed that pertaining to static resistance limits.

In order to ascertain the range of validity of the diagrams of Fig. 7, two different sets of conditions of load application should be considered—(1) loads inducing strain rates that exceed the rate of permanent deformation (rate of yield) of the material and (2) loads inducing lower strain rates. The fact that, in the latter case, the yield point cannot be exceeded furnishes the criterion which defines a critical number of load cycles below which the resistance under conditions of static strain is lower than that under conditions of rapid load repetition.

The picture of the dynamical resistance of metals would be incomplete without the inclusion of the concept of "damage" by overstrain which has considerable bearing on the safety of structures. The "damage range"[21,22] expresses the extent of damage with regard to the endurance limit which is caused by incidental overstrain. The "damage line" delimits this range by relating the amplitude of overstrain that can be sustained by the material without which the final endurance limit is affected to the maximum permissible number of repetitions of this overstrain cycle (Fig. 8). Information about this phenomenon is scarce and refers to conditions of total load reversal only.

The capacity of a structural material to sustain occasional overstrain without damage to its ultimate endurance limit is very important from a design point of view. According to the previously stated stipulation, the design live load should provide for service conditions of reasonably high frequency of occurrence, whereas less probable, and therefore less frequent, extreme loading conditions should be covered by the factor of safety. If the material is able to carry this occasional excess of extreme loads over the design load by its capacity to sustain occasional overstrain without damage to its ultimate endurance limit, this limit may be directly correlated with the design load without introducing any margin of safety that would otherwise be necessary to cover the fluctuations of the load intensity above the design value specified.

Resistance to "damage" by overstrain of structural materials should be made an urgent object of research. The results of such research will most certainly have some influence upon the manufacture of such materials, since it will reveal the relation of "damage sensitivity" and the general mechanical properties. Structural metals can have "damage lines" of different shapes but still have similar standard properties, such as static strength, elongation, and endurance limit. In Fig. 8 line a characterizes a material with a capacity to sustain frequent overstrains of moderate intensity, line b a variety of the same material with a capacity to sustain considerable overstrain if its frequency of occurrence is low, line c a variety with a remarkably high resistance against "damage" by frequent and high overstrain. The endurance limit as a function of the number of load repetitions is the same for all three varieties; but their performance will be different and will depend on the frequency of overstrain

[21] "Damage and Overstress in the Fatigue of Ferrous Materials," by H. W. Russel and W. A. Welcker, *Proceedings*, A.S.T.M., 1936, Pt. II, pp. 1118–1135.

[22] "Effect of Overstressing and Understressing," by J. B. Kommers, *ibid.*, 1938, Pt. II, pp. 249–268.

and its amplitudes. Every one of the three varieties of the material is suited to a particular character of load fluctuation.

Like all physical properties the resistance values of structural materials are constants in a statistical sense only; the level of control enforced in all stages of the manufacturing process determines the character of the distribution function. In Fig. 3 the distribution functions for the yield point and the tensile strength of the silicon steel manufactured for the Golden Gate Bridge has been drawn. The practical range of chance fluctuations reaches about 15% below the mean or about 12% below the modal value. Concrete is less uniform than steel; investigations[23,24] have shown maximum ranges of fluctuations of individual values of compressive strength of concrete manufactured on the site under normal control to be approximately 35% about the mean; this range is reduced to one half by control under laboratory conditions.

### Fluctuations and Uncertainty in the Dimensions

The ranges of variation of the sectional resistance values can be derived from the tolerances of the linear dimensions. Since these tolerances do not generally exceed 2% to 3% the fluctuations of areas and section moduli are within the range of 4% to 6% about the mean.

Design as a process of prediction, however, is not concerned only with the initial state of the structure but also with its change in the future owing to deterioration and corrosion. These influences must be considered, therefore, in the computation of the initial safety factor.

A more detailed treatment of all influences is included in the original manuscript which has been placed on file for reference in the Engineering Societies' Library[24a] in New York, N. Y.

### Application of Methods Developed

The method of evaluating the appropriate safety factor will be illustrated and its practical application demonstrated by computing the working stresses to be used in the design of stringers and floor beams of highway bridges of low-alloy (silicon) steel.

In order to derive working stresses or safety factors it is necessary to establish a uniform basis of reference since both concepts have a definite meaning only in connection with a specified service load. The evaluation of this load must be the first step in any analysis of this kind. It requires the computation of the probabilities of occurrence of different groupings of pertinent load units with reference to the definition of an adequate design live load.

Under the assumption that one fourth of all vehicles are trucks or units of similar load intensity, and that only the heaviest types circulating at present are considered (see conclusion 2 under the heading "Live Load: Intensity and Distribution of Live Loads for Bridges"), the frequency of occurrence of trucks among a total number $n = t\,m$ of vehicles (in which $t$ denotes the number of

[23] Discussion by C. E. Arnold of "Manufacturing Concrete of Uniform Quality," by William M. Hall, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), pp. 1367–1371.

[24] "Betondruckfestigkeit," by D. Bendel, Rept. of Cong. for Reinforced Concrete, Liege, Belgium, 1930, p. 238.

[24a] 29 West 39th St., New York, N. Y.

traffic lanes and $m$ the number of vehicles per lane) can be derived by the binomial law and approximated by the Gram-Charlier series. The frequency distributions for $n = 6$, 12, 24, and 96 vehicles,[24b] respectively, have been presented in Fig. 9. With the aid of Eqs. 23 and 25 and introducing $v = 11.5$ miles per hr as the "critical" speed of the design load, the design load intensities —including the dynamic effect $w_{lo}(1 + I_o)$ pertaining to the respective modes

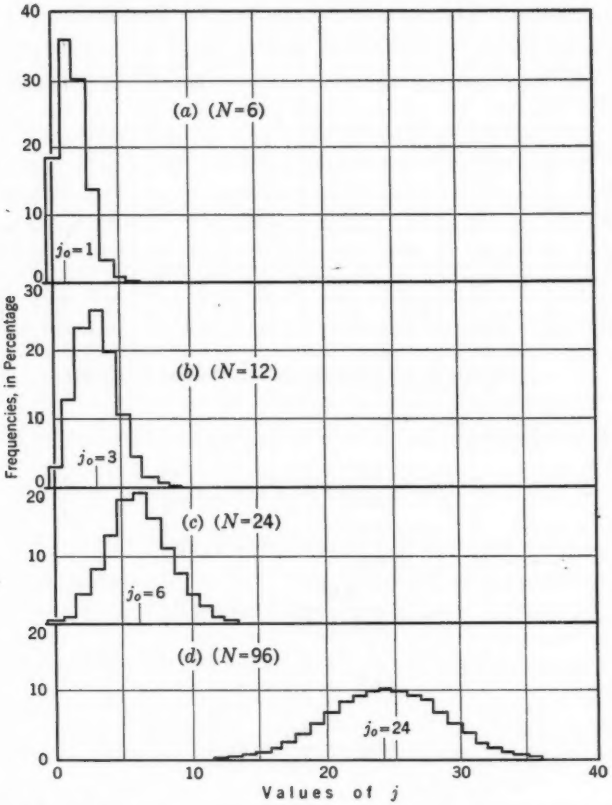



FIG. 9.—FREQUENCY DISTRIBUTION OF THE NUMBER OF TRUCKS COUNTED IN A TOTAL NUMBER $n$ OF VEHICLES

of the distribution functions as well as the ranges of fluctuation of those functions pertaining to the probability $P_s < 0.001$ of values to fall outside this range—can be established as shown in Table 1.

For short spans the passing of a single truck per lane traveling at maximum speed may produce higher design load intensities than those given in Table 1. A direct comparison of load intensities would be misleading, however, because the difference in the ranges of fluctuation for the two types of loading results in different values of the safetv factor to be applied.

[24b] Correction for *Transactions:* In Fig. 9 change "N" to "n" in four places.

Introducing the bending moment as a strain characteristic

$$s_a = M = C_1 w_d l^2 + C_2 (1 + I) w_l l^2 \quad \ldots\ldots (27)$$

and as resistance characteristic the product of section modulus $S$ and strength of the steel at failure $s_f$

$$s_r = S s_f \quad \ldots\ldots (28)$$

TABLE 1.—Design Live Loads per Lane and Ranges of Fluctuations[a]

| Number of vehicles, $n$ | Number of Trucks | | Design Values | | | Maximum $w_l$ (lb per ft) | Maximum Fluctuation (%) | | | Loaded Length (Ft) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Modal | Maximum | $w_{lo}$ (lb per ft) | $1 + I_o$ | Col. 4 × Col. 5 (lb per ft) | | $+\Delta w_{lo}$ | $+\Delta w_l$ | $+\Delta(1 + I)$ | $t = 1$ | $t = 2$ | $t = 4$ | $t = 6$ |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) |
| 6 | 2 | 5 | 585 | 1.48 | 865 | 925 | 25 | 58 | 28 | 240 | 120 | 60 | 40 |
| 12 | 3 | 8 | 510 | 1.33 | 680 | 830 | 25 | 63 | 22 | 480 | 240 | 120 | 80 |
| 24 | 6 | 14 | 510 | 1.24 | 630 | 770 | 25 | 51 | 17 | 940 | 470 | 235 | 117 |
| 96 | 24 | 36 | 505 | 1.12 | 565 | 745 | 25 | 47 | 10 | 3,800 | 1,900 | 950 | 475 |

[a] Ranges of fluctuation pertain to probability $P_s < 0.001$.

Applying Eqs. 17 and 18:

$$\left.\begin{aligned} a_1 &= \frac{\partial M}{\partial w_d} = C_1 l^2 \\ a_2 &= \frac{\partial M}{\partial w_l} = C_2 (1 + I_o) l^2 \\ a_3 &= \frac{\partial M}{\partial (1 + I)} = C_2 w_{lo} l^2 \end{aligned}\right\} \quad \ldots\ldots (29)$$

and

$$\left.\begin{aligned} (a_1)' &= \frac{\partial s_r}{\partial S} = s_{fo} \\ (a_2)' &= \frac{\partial s_r}{\partial s_f} = S_o \end{aligned}\right\} \quad \ldots\ldots (30)$$

Hence

$$s_a = s_{ao}\left(1 + \frac{\Delta s_a}{s_{ao}}\right) = [C_1 w_{do} l^2 + C_2 (1 + I_o) w_{lo} l^2] \times \left[1 + \frac{1}{1 + \dfrac{C_1}{C_2}\dfrac{w_{do}}{w_{lo}(1 + I_o)}}\right] \times \sqrt{\left(\frac{C_1}{C_2}\right)^2 \left(\frac{w_{do}}{w_{lo}}\right)^2 \left(\frac{\Delta w_d}{w_{do}}\right)^2 + \left(\frac{\Delta w_l}{w_{lo}}\right)^2 + \left[\frac{\Delta(1 + I)}{(1 + I_o)}\right]^2} \quad \ldots\ldots (31)$$

and

$$s_r = s_{ro}\left(1 - \frac{\Delta s_r}{s_{ro}}\right) = S_o s_{ao}\left[1 - \sqrt{\left(\frac{\Delta S}{S_o}\right)^2 + \left(\frac{\Delta s_a}{s_{ao}}\right)}\right] \quad \ldots\ldots (32)$$

For the freely supported short-span structural members considered (stringers and floor beams) the respective design load intensities are produced by one truck per lane traveling at a maximum speed of 50 miles per hr. The ratios $\frac{w_{do}}{w_{lo}}$ may be assumed to vary between 0.1 (for stringers) and 0.4 (for heavy floor beams). With such comparatively small effects of the dead load, the strain may be assumed to fluctuate between zero and a maximum value. The total number of strain cycles generally will reach, or even exceed, the two-million mark for bridges carrying main highways, the rate of strain induced by the rapidly moving load being in the vicinity of 0.3% per sec in structures of low-alloy steel. Since this rate exceeds considerably the intrinsic plastic flow rate of steel of about 0.05% per sec, failure will be brittle, and the structure's resistance will be a function of the ultimate tensile strength $s_t$ of the material. For a load oscillating between zero and a maximum value, numbering a total of more than two million cycles, the actual strength of riveted structural parts (floor beams) is $s_{fo} = 0.4\ s_{to}$ and of plain members (stringers) is $s_{fo} = 0.45\ s_{to}$ (see Fig. 7).

Let: $\frac{\Delta w_l}{w_{lo}} = 0$ for one-lane and two-lane highways on which the design load is equal to the maximum load; $\frac{\Delta w_l}{w_{lo}} = 0.58$ for six-lane highways (see Table 1); $\frac{\Delta w_{do}}{w_{do}} = 0.1$ (in which, see Eq. 22, $w_{do} = 1.1\ \bar{w}_{do}$); $\frac{\Delta(1+I)}{1+I_o} = \frac{1.00}{(1+I_o)\sqrt{n}}$ (see Eq. 26); $\frac{\Delta S_a}{S_a} = 0.06$; and $\frac{\Delta s_t}{s_{to}} = 0.12$ (see Fig. 3). For free supports $C_1 = C_2$. Inserting the foregoing values in Eqs. 5, 31, and 32, the ranges $\Delta s_a$ and $\Delta s_r$, the safety factor $\zeta$, and the allowable stresses $\frac{1}{\zeta} S_{fo}$ are found to be:

For stringers ($t = 1$)—

$\Delta s_a = +0.40\ s_{to}$; $\Delta s_r = -0.14\ s_{ro}$; $\zeta = 1.64$; allowable stress $s = 0.275\ s_{to}$

for floor beams ($t = 2$)—

$\Delta s_a = +0.32\ s_{to}$; $\Delta s_r = -0.14\ s_{ro}$; $\zeta = 1.54$; allowable stress $s = 0.260\ s_{to}$

for floor beams ($t = 6$)—

$\Delta s_a = +0.51\ s_{to}$; $\Delta s_r = -0.14\ s_{ro}$; $\zeta = 1.75$; allowable stress $s = 0.229\ s_{to}$.

The comparatively high value of the safety factor for six traffic lanes is due to the fact that for six vehicles the design load intensity is no longer represented by the maximum load intensity (see Fig. 9 for $n = 6$).

The data utilized in this example are not elaborate enough to enable one to evaluate the probability of failure pertaining to the foregoing safety factors. A rough estimate becomes possible, however, if the distribution functions of the constituents $w_l$ and $s_t$ are assumed to be representative of the frequency distributions of the resultant characteristics $s_a$ and $s_r$. For the six-lane floor beam $P_s < 6\ (\frac{1}{4})^5\ (\frac{3}{4}) = 0.005$ and $P_r < 0.0001$. Therefore, the probability

of failure $P_f < 1/2{,}000{,}000$ as required for an expected number of load cycles which slightly exceeds 2,000,000.

In order to compare the foregoing with values specified in actual design a common base of reference is found in the "reduced" permissible stress $s'$, which is the stress permissible under the action of dead load $\bar{w}_{do}$ and the static weight of the maximum live load $w_l$ (max), if the specified safety of the design is to be maintained. Hence, for a uniformly distributed live load and freely supported span

$$s' = s \frac{\bar{w}_{do} + w_l \text{ (max)}}{w_{do} + (1 + I_o) w_{lo}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (33)$$

In the design of the Golden Gate Bridge, for instance, the frequency distribution of the tensile strength $s_t$ (Fig. 3) of silicon steel was established; and the permissible stress specified for stringers and floor beams was 22,000 lb per sq in. to be applied with an impact increment of 50% of the live load for stringers and 25% for floor beams. For the latter, a reduction of 25% of the live load was intended to allow for the large number of traffic lanes ($n = 6$).[25] These stipulations lead to the following values for the reduced permissible stress: For stringers—$s' = 0.69 \times 22{,}000 = 15{,}000$ lb per sq in.; and for floor beams—$s' = 1.04 \times 22{,}000 = 23{,}000$ lb per sq in. The reduced permissible stresses resulting from the present analysis of the safety factor can be computed by introducing the mode of the frequency distribution (Fig. 3), $s_{to} = 87{,}000$ lb per sq in. Hence: For stringers—$s' = 0.44 \times 0.275 \times 87{,}000 = 10{,}500$ lb per sq in.; and for floor beams—$s' = 1.35 \times 0.229 \times 87{,}000 = 27{,}000$ lb per sq in.

Comparison of the foregoing values shows that even a complex design (such as that of the Golden Gate Bridge, which empirically provides for the different probabilities of occurrence of maximum load intensity in different structural parts) will not yield adequately balanced stresses and dimensions as long as the safety factor is being "selected" instead of computed by rational and objective methods. It appears that the permissible stress specified is too high for the stringers on Golden Gate Bridge, and unduly low for the comparatively long-span floor beams.

## PART 3.—CONCLUSIONS

The safety factor is an objective value of correlation between actual strain and potential resistance that can be ascertained by the rational methods suggested in the paper. Loads, external conditions, and pertinent physical properties and phenomena are analyzed under the common aspect of their relation to the magnitude of the safety factor. This factor is intended to provide for relatively infrequent loading, and other conditions, whereas the design itself considers conditions of relatively frequent occurrence. The computation of the objective minimum safety factor and the consequent permissible stresses is in a balanced design of no less importance than the determination of the principal static characteristics.

[25] "The Golden Gate Bridge," Rept. of the chf. engr. to the board of directors of the Golden Gate Bridge and Highway Dist., Schwabacher-Frey, San Francisco, Calif., 1937, p. 81.

Since the safety factor, expressing the excess of possible, or potential, resistance over the computed strain to be provided in order to prevent failure with a reasonably high probability, derives its existence from the imperfection inherent in the performance of any human action and the uncertainty marking the results, as well as from the acknowledged ignorance about certain phenomena, and since it relates computed strain to actual resistance, it can refer to the design as an entity only and not to different features of it. Therefore, it can have only one value for a certain design. Individual safety factors for different influences, such as for dead load or for live load, separately, being inconsistent with the basic concept, can have no real meaning.

The current specifications and standards applying to design loads and secondary influences, as well as strength and permissible stresses, should be analyzed as to their correspondence to reality, and the objective conditions should be ascertained as a base for new and more adequate specifications. Conventional specifications seem too lenient as far as short-span structural members are concerned, and unnecessarily severe for structural members and structures of medium and long span. The use of adequately balanced permissible stresses in the different elements of a structure will solve, automatically, the problem of the occasional passage of exceptionally heavy loads, raised in connection with the transport of military equipment with very heavy axle concentrations.[26]

In spite of extensive experimental research concerning a great number of structural qualities, lack of adequate information about certain basic aspects of design calls for urgent attention and an effort to make up the deficiencies. Some of the most important effects, such as that of the rate of the induced strain, or the capacity of the material to sustain occasional overstress, have been almost neglected. The evidence applying to other phenomena and qualities which have actually been investigated has generally not been collected with regard to subsequent statistical interpretation, and therefore needs amplification.

By adopting the principle that neither design loads nor safety factors and permissible stresses should be specified arbitrarily, it will be possible not only to eliminate inadequate design, but frequently to achieve considerable economy. It will be possible, moreover, to determine correct safety factors and permissible stresses for unconventional structures or new structural forms and materials.

[26] *Civil Engineering*, February, 1941, p. 116.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

# DESIGN DEVELOPMENTS—STRUCTURES OF THE TENNESSEE VALLEY AUTHORITY

## A SYMPOSIUM

PAGE

Structural Features of Hydraulic Structures.
By Ross M. Riegel, M. Am. Soc. C. E. ..... 1194

Design of Hydroelectric Stations.
By George P. Palo and Constant R. Marks, III, Assoc. Members, Am. Soc. C. E. ..... 1209

Civil Engineering Features of Steam Plant at Watts Bar.
By Adolf A. Meyer, M. Am. Soc. C. E. ..... 1224

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by March 1, 1946.

# STRUCTURAL FEATURES OF HYDRAULIC STRUCTURES

By Ross M. Riegel,[1] M. Am. Soc. C. E.


## Synopsis

Some of the more important or unusual features of structural design of the dams and other hydraulic structures constructed by the Tennessee Valley Authority (TVA) are treated in this paper. The design of power plants is discussed in the remaining two papers. The hydraulic performance of such structures is beyond the scope of the Symposium. This paper is confined to a few principles and methods governing the design of the structures themselves—and to only those which are considered to have some degree of novelty. Some evidence or argument supporting such procedures is presented. The principal hydraulic structures of the TVA, of course, are the dams. As a background for the treatment of this subject Table 1, showing the location and the more important features of these dams, is presented.


## Concrete Dams

It is held to be a fundamental principle of dam design that an impervious surface should be presented to the impounded water and that provisions should be made for the escape to tailwater of any seepage or leakage that may pass the tight surface or diaphragm.

The two first dams of the TVA (Norris and Wheeler) were designed by the Bureau of Reclamation, U. S. Department of the Interior, as gravity dams and the original construction plant was assembled to be suitable for this type of structure. Efforts have been made by the TVA to transfer construction equipment from job to job. For this reason there were certain advantages in adhering to the same general types of structure from job to job. Under general conditions on these projects, form costs tended to be high, whereas the cost of materials for mass concrete tended to be low, and such comparison of costs as has been made in general showed little difference in cost between gravity dams and other types of structure. Structures of the mass type present less surface to the action of weathering agencies than do those of the reinforced type, for example. The massive TVA structures are made of excellent concrete and future maintenance expense on account of any surface deterioration of the concrete may be expected to be less for the mass structure than for the reinforced one. For these reasons the concrete dams of the TVA have been designed as gravity structures. In accordance with the principle enunciated in the preceding paragraph, the effort has been made to obtain a surface as tight as possible at the upstream face. In the larger dams the upstream face has been constructed with an enriched concrete. In the smaller structures, a richer concrete has been used throughout.

[1] Head Civ. Engr., Design Dept., TVA, Knoxville, Tenn.

*Foundation Treatment and Uplift.*—Considering the foundation as an essential part of the structure, elaborate cutoff grouting systems have been specified with a view to preventing seepage and uplift beneath the structures. Holes at

TABLE 1.—Locations and Principal Dimensions (in Feet) of the Dams of the Tennessee Valley Authority

| Dam | Concrete Section (Including Powerhouse) | | | Embankment Section | |
|---|---|---|---|---|---|
| | Length | Height[a] | Power Head[a] | Length | Height[b] |
| Kentucky | 1,823 | 206 | 57 | 6,800 | 60 |
| Pickwick Landing | 2,050 | 113 | 63 | 5,700 | 55 |
| Wilson | 4,862 | 137 | 96 | .... | .... |
| Wheeler | 6,342 | 72 | 53 | .... | .... |
| Guntersville | 1,384 | 95 | 45 | 2,600 | 46 |
| Chickamauga | 1,410 | 129 | 55 | 4,400 | 46 |
| Watts Bar | 1,739 | 97 | 60 | 1,200 | 57 |
| Ft. Loudoun | 1,285 | 122 | 80 | 2,600 | 55 |
| Norris | 1,570 | 265 | 194 | 290 | 50 |
| Cherokee | 1,697 | 175 | 148 | 5,100 | 60 |
| Douglas | 1,682 | 202 | 128 | .... | .... |
| Fontana | 2,330 | 480 | 430 | .... | .... |
| Hiwassee | 1,287 | 307 | 254 | .... | .... |
| Apalachia | 1,183 | 150 | 440[c] | .... | .... |
| Chatuge | .... | .... | .... | 2,850 | 140[a] |
| Nottely | .... | .... | .... | 2,300 | 184[a] |
| Ocoee No. 3 | 610 | 110 | 316[c] | .... | .... |

[a] Maximum. [b] Average. [c] Total head by diversion tunnel.

a spacing as close as 2 ft have been drilled for this purpose. This grouting should be effective for many years and, in the course of time, it should become reinforced by the deposition of sediments in the reservoir bottom.

On the main river dams the foundations generally have consisted of calcareous rocks with approximately horizontal bedding considered favorable to the accumulation of uplift pressures in the foundation itself. For this reason

these dams, as well as those on tributaries, have been provided with a definite drainage system consisting of vertical holes 8 ft to 10 ft apart, drilled 30 ft to 40 ft into rock downstream from the grouted cutoff (see Fig. 1). The purpose of these drains is to prevent the accumulation of any considerable upward pressure in the foundation rock. In addition to the foregoing, liberal drainage has been provided under the aprons of the spillways, including the drilling of vertical

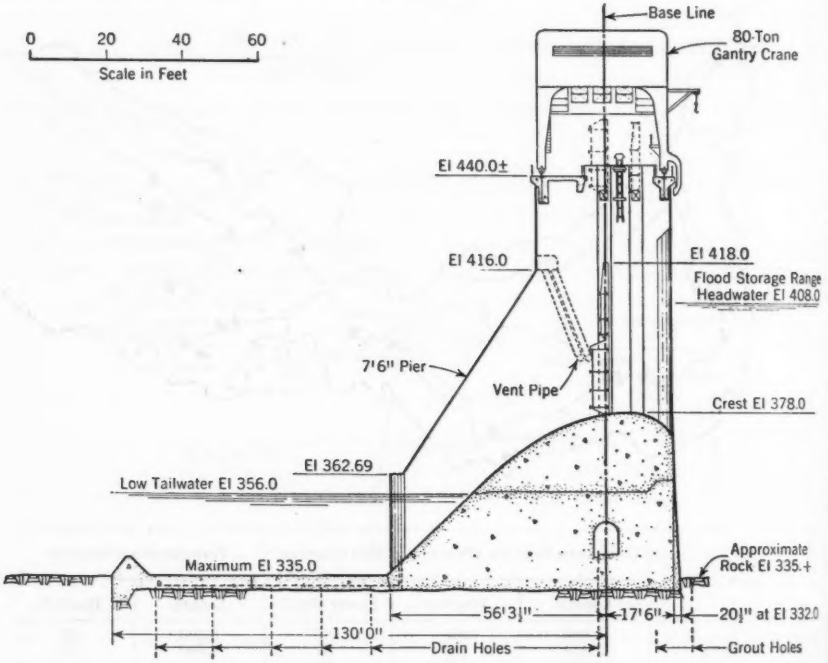



FIG. 1.—SECTION THROUGH SPILLWAY, PICKWICK LANDING DAM

holes to tap any horizontal openings that might exist in the foundation. Although these were provided primarily for the protection of the apron itself, they contribute to the maintenance of low uplift pressures under the main dam.

Having provided the drains, it was logical to capitalize upon their presence. Accordingly, in the main river dams, which are of moderate head, the uplift assumption in the foundation for design purposes was restricted to tailwater pressures downstream from the line of drains. Uplift pressures would increase uniformly from tailwater pressure at the drains to headwater pressure at the upstream face of the dam. Pressures were assumed to be applied over two thirds of the area of the base of the dam. In these cases, as elsewhere, such resistance to horizontal movement as the apron might offer was neglected.

For the higher structures such as Hiwassee, Douglas, Cherokee, and Fontana (Fig. 2), it was felt that the foregoing rule was not quite conservative enough even with the close drainage system provided. In these cases the uplift assumption was modified so as to assume one half of headwater pressure at the

line of drains—the pressure varying in a straight line to headwater and tailwater, respectively. The assumed area of two thirds of the base was retained.

As to the uplift effects in the concrete itself, concerning which there is considerable uncertainty, TVA engineers have assumed that the open contraction joints, which have been used uniformly, are in themselves relief points for uplift pressures. Inspection galleries would also be relief points. However, in the higher dams where there is an excess of 100 ft of head, drainage wells were built in the dam itself, substantially on the same line as the foundation drains, to

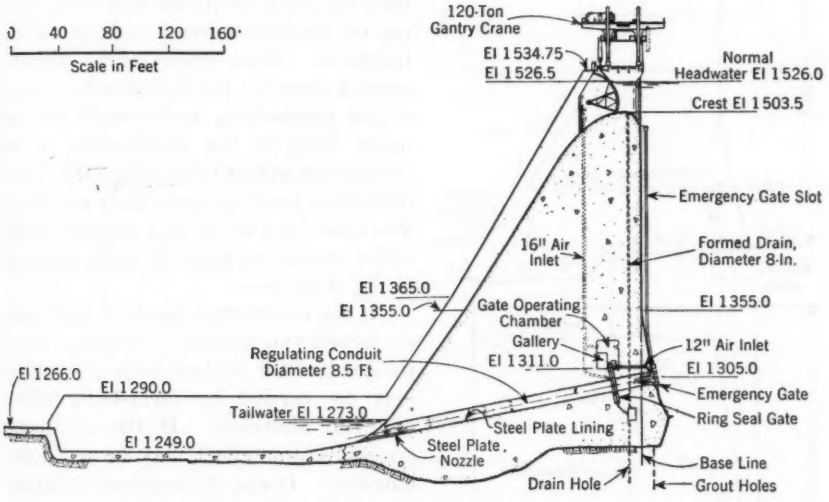



Fig. 2.—Section through Spillway, Hiwassee Dam

provide a more definite relief for possible uplift accumulation in the concrete than would be afforded by the joints and galleries alone. Pressure distribution is assumed the same as that for the foundation.

*Measurements of Uplift.*—On Hiwassee, Douglas, Fontana, and Cherokee dams, a number of devices were installed to measure uplift in the foundation in the vicinity of the foundation contact. Fig. 3 shows a typical observation of this kind on Hiwassee Dam. Similar diagrams were prepared for three other stations in this dam as well as for similar stations in the other two dams, and they all have the same general character. The uplift pressure varies more or less uniformly from headwater to the drainage line where it is a minimum; between this drainage line and the toe of the dam there is a moderate rise and subsequent fall to the toe. Inspection of the drains and frequent measurements since construction have shown that this condition is stable and well within the design assumptions for the structures. Inspection also has shown that the drain outlets from the foundation continue to flow at a moderate rate. There is continuing evidence that the foundation drainage systems are functioning as intended.

The record of certain measurements of the pressure of pore water in the concrete at Hiwassee Dam, at various distances from the surface of the con-

crete, is presented in Fig. 4. Pressures were detected in 1943 at a depth of 12 ft. A gradual penetration of reservoir water is indicated. Of course, these observations indicate nothing as to the effective area of application of such pressures. (It is to be noted that this water passes an enriched surface layer of concrete.) The vertical drains, however, should limit the accumulation of pressure.

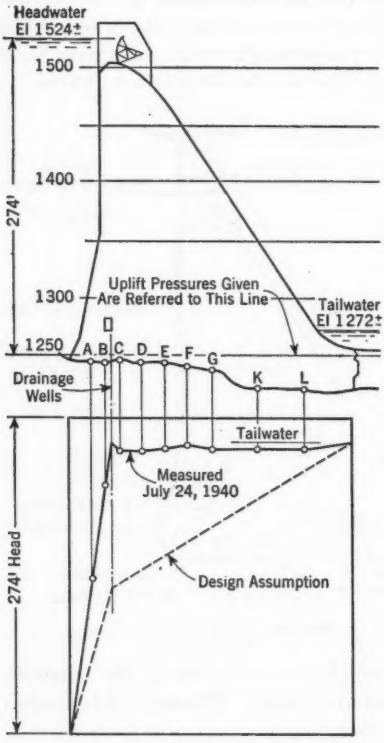



FIG. 3.—UPLIFT PRESSURE AT THE BASE OF HIWASSEE DAM (BLOCK NO. 11)

*Contraction Joints.*—The general practice of the TVA is to provide free contraction joints in its gravity dams, relying on duplicate metal seals for watertightness. With open joints, there is general freedom for individual blocks to adjust themselves, under dead load and water load, to the peculiarities of the foundation without throwing part of their individual burdens upon their neighbors. The open joint serves as a drainage outlet and a means of relief of uplift pressures in the concrete.

If the contraction joints in high dams are keyed and grouted, certain parts of the load at the highest part of the dam may be carried by shear-twist effects into the abutment. If the abutments are steep, this effect may be quite pronounced. It may be analyzed by a trial-load method, but it seems preferable to design the structures so that the shear-twist effect will be avoided and no allowance made for it. In general, therefore, the gravity dams of the TVA are designed with completely free transverse joints.

In the two highest dams—Fontana and Hiwassee—this plan has been modified as follows: Keys and grouting have been provided in a limited portion of the transverse joints at the bottom of these dams. At Hiwassee Dam this amounts to about 25% of the height of the joint, and at Fontana Dam the height averages somewhat less than this proportion. This limited amount of grouting has been a concession to the belief of some members of the TVA staff in the practice of grouting. The most important justification in the writer's view is that the grouting of the joints at the bottom has a strong tendency to cure structural cracks in the concrete which tend to develop in the region of high restraint adjacent to the foundation. Although these cracks are subject to autogenous healing, if they are small, larger cracks should be penetrated by grout from the joints and be cured in this way. With limited grouting at the bottom of the joints, most of the dam section would still be free to deflect and the shear-twist effects would still be minimized.

*Foundation Restraint.*—Foundation restraint is taken seriously by TVA designers. Since concrete develops temperature in setting and then loses its temperature by radiation as setting is completed, volume must necessarily tend to change. At the foundation level these changes result in tensile stresses and the concrete tends to crack. It is general experience that a spacing of contrac-

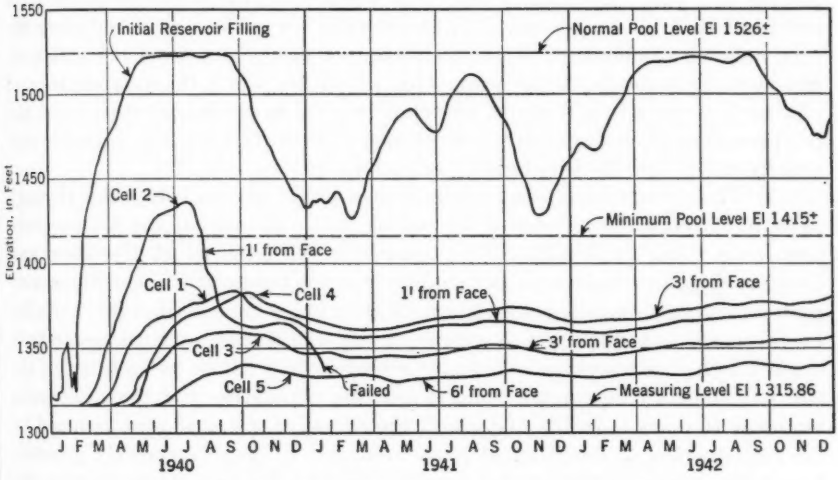



FIG. 4.—MEASURED PORE PRESSURE IN THE BODY OF HIWASSEE DAM

tion joints of not much more than 50 ft will produce structures in which cracks transverse to the axis are not material. Nevertheless, cracks parallel to the axis tend to be frequent in the gravity dam and are considered deleterious.

The most rational approach to the prevention of these cracks is to control the manufacture of concrete so as to reduce the maximum temperature of the concrete and the subsequent drop of temperature as much as practicable in the region of high restraint. Three methods to this end have been followed in the TVA organization: (1) The use of low-heat cement and a low cement content; (2) pouring concrete slowly to minimize the temperature rise in the region of restraint; and (3) circulating cold water through pipes to control the temperature rise.

(1) Lower maximum temperatures are obtained in the mass concrete when low-heat cement is used than would be realized under parallel conditions with standard or modified Portland cement. It is also the general practice of the TVA in major structures to use a maximum aggregate size of 6 in. with four grades of coarse aggregate and sometimes two grades of fine aggregate, all proportioned by weight. With such proportioning and a water-cement ratio of about 0.75 by weight, a very large amount of concrete has been made with about 0.8 bbl of cement per yard. This concrete is entirely adequate from the strength standpoint, whether low-heat cement or other grades have been used, and the low cement content causes low maximum temperatures, advantageous from the standpoint of restraint.

(2) Temperature rise in the region of restraint is minimized by slow pouring. By this is meant the placing of thin pours at a considerable time interval so as to permit the radiation of a material portion of the heat of hydration from the top of a pour before the succeeding pour is placed. This measure is of most importance in hot weather and near the foundation. It involves usually the placing of a pour, say 2.5 ft thick, with a time interval of two or three days before placing the next pour. This method is very effective in reducing the accumulated temperature rise and subsequent drop. Its necessity will vary considerably throughout the year. In the winter when the temperature of placing the concrete is likely to be below its final mean temperature, and the temperature changes are less in magnitude, this procedure is less valuable and usually can be modified materially, or eliminated.

(3) The rise of temperature can be limited by circulating cold water through pipes in the concrete to remove the setting heat and prevent the temperature from ever rising materially. This procedure was followed at Hiwassee and Douglas dams in certain local situations where the temperature conditions were particularly unfavorable, but on Fontana Dam it has been used systematically. In this case, where the dam is approximately 450 ft high and the concrete is subject to high stresses, extraordinary attention was felt to be justified in the effort to control temperature changes near the foundation and the consequent tendency to crack. In the first place, the effects of restraint were reduced by the introduction of longitudinal joints designed to be subsequently grouted. Low-heat cement could not be obtained for the Fontana Dam, but in the region of restraint adjacent to the foundation both the slow pouring method and that of artificial cooling were resorted to in the warmer weather. Concrete placed in this region in the hot months of the summer was cooled by refrigerated water at 40° F to 45° F, passing through pipes 2.5 ft, center to center. By these combined means the temperature rise was kept at about 15° F (see Fig. 5). In the region above the foundation, where restraint became unimportant, the concrete was allowed to rise in temperature and then was cooled by circulating water to approximately its final mean temperature so as to permit the effective grouting of the longitudinal joints.

All the foregoing discussion relates to mass concrete. Sometimes there are relatively thin structures, such as spillway piers, which are subject to restraint at the foundation or at the point of connection with larger structures. The tendency to crack is the same. However, in such cases it has been deemed preferable to control cracks by reinforcement rather than to endure the delay and complications caused by slow pouring and artificial cooling. Moreover, the durability requirements of such elements generally preclude a low cement content. Progressive observations on certain structures and office calculations finally led to a standard provision that, in the region adjacent to the restraint, about 0.75% of steel should be used to prevent or distribute cracks. In the thicker walls and piers, this was reduced to 0.50% of the outer 2 ft. As the height of the structure increases above the foundation, the restraint diminishes and the proportion of reinforcing steel is much reduced. Research and observation led to the conclusion that, at a height above the foundation equal to one

half the length of the structural element in question, there is no longer any restraint, provided that the concrete is poured without any long interruption.

*Stresses in Concrete Dams.*—Some comment on stresses in gravity dams is pertinent. It has been the practice of the TVA to calculate stresses on the conventional straight-line basis. The weight of the concrete is determined as accurately as feasible and is used in calculations. Uplift assumptions have been made as outlined previously. The requirement has been made that no tension

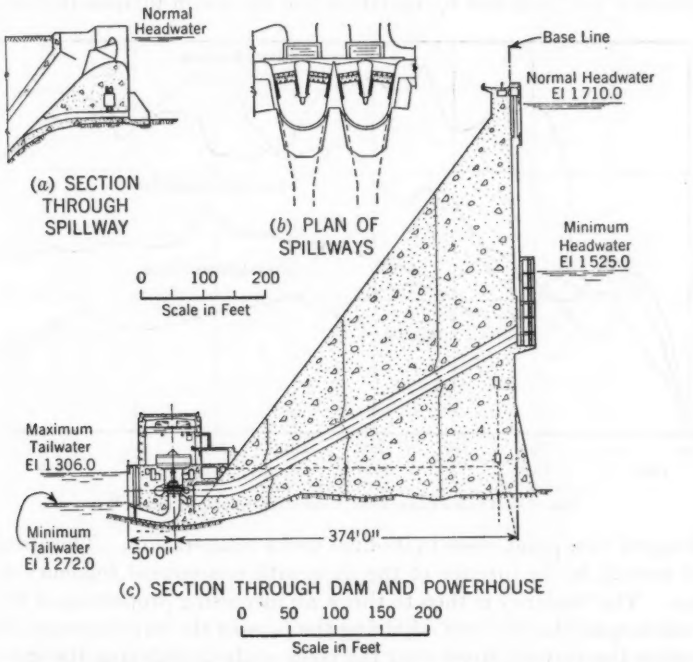



FIG. 5.—THE FONTANA PROJECT (TURBINES, 91,500 HP AT 325-FT HEAD; GENERATORS, 67,000 KW)

should exist at the upstream face under the conventional calculation and a somewhat variable margin over this requirement has been assumed. Some attention has been paid to the practices of other organizations as to the major dimensions so that the determination of such things as the ruling downstream slope of the dam has not been based exclusively on calculations. Fontana Dam was made somewhat heavier in section than Hiwassee Dam largely because it is higher.

The TVA approach has been considerably influenced by the results of research conducted on Norris and Hiwassee dams for several years. In these structures a number of Carlson measuring instruments were installed for the purpose of measuring strains, stresses, joint openings, temperatures, etc. Fig. 6 shows some results at three measuring stations as reported by Douglas McHenry.[2] This diagram covers a period of four and one-half years, including the time of filling the reservoir, and of more than three years of subsequent

[2] "Measurements of the Structural Behavior of Norris Dam," by Douglas McHenry, *Technical Monograph No. 53*, TVA (mimeographed bulletin).

fluctuation. The two most striking points about this diagram are the way in which the vertical strain persists near the upstream face of the dam with increasing water load and time while the strain at the midpoint diminishes. Strains near the downstream face generally rise with the increase of load and the passage of time, as might be expected.

There appear to be two reasons for these marked departures from the conventional straight-line variation. During the period of observation the heat of hydration has been lost by radiation and the mean temperature of the dam

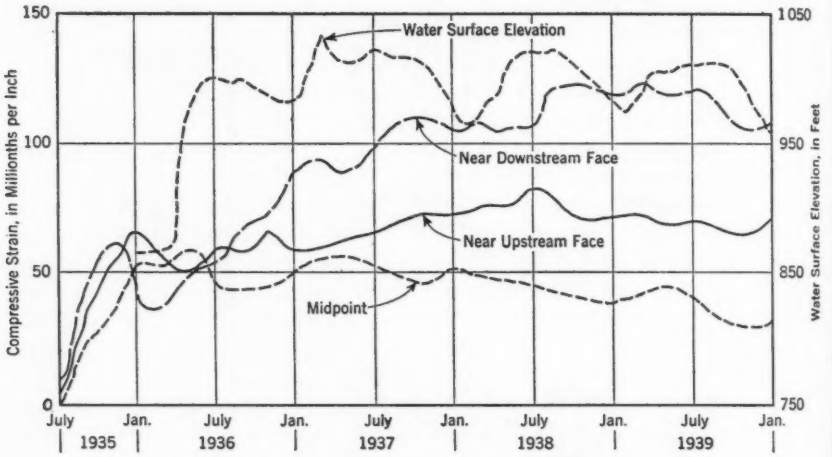



FIG. 6.—MEASUREMENTS OF VERTICAL STRAIN, NORRIS DAM

has dropped to a point close to its final mean temperature. This drop has occurred mainly in the interior of the dam with consequent tendency to loss of volume. The tendency is then to throw an increasing proportion of the weight of the dam upon the concrete adjoining the faces of the structure and, therefore, to increase the vertical stress near the faces while diminishing the stress in the center of the dam. Mr. McHenry also points out that the lower part of the upstream face of Norris Dam has a considerable batter, namely 1 : 5, and that thereby a considerable vertical load is applied to the heel of the dam.[2] He believes that this water load is largely transmitted into the foundation directly, without ever being diffused in its effect over the structure as a whole. This theory seems plausible but is certainly not in accord with the conventional method of stability analysis. More recent analyses of similar conditions at Hiwassee suggest that the higher heel stress is more probably traceable to the penetration of the concrete by reservoir water, combined with the temperature load transfer.

Fig. 7 records some tentative stress distributions at Hiwassee Dam, based, in this case, on data from five measuring stations. At each station strains are measured in five directions and correlated, and deformation of free cylinders under no stress is also measured. The effects of temperature change as well as load are reflected in the measurements. The phenomena are complex, as are the interpretations of them, and the diagram is tentative as to actual stress values.

The five sets were placed near the foundation in a block that was discontinued for several months, at a level a few feet above the instruments, while the river was diverted over it and cooled it. Then the pouring of the block was resumed and carried to full height. Fig. 7(*b*) shows the same high stress at the heel under water load that was found at Norris Dam. Later measurements show a drop in stress near the third points and an increase at the faces, par-

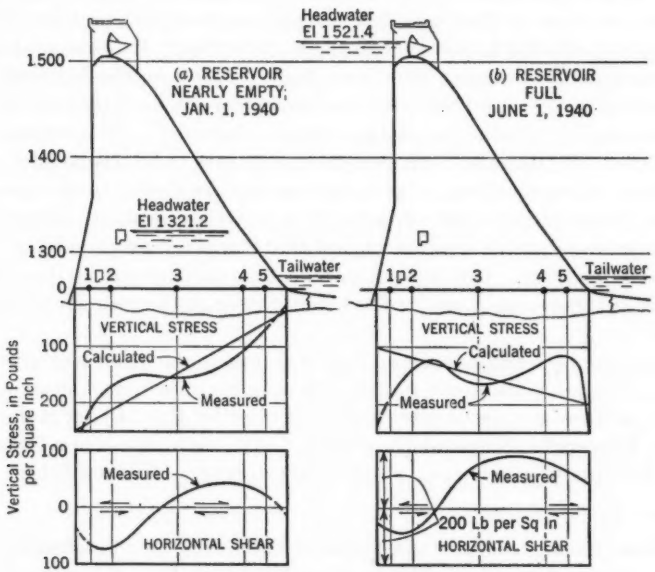



FIG. 7.—MEASURED STRESS DISTRIBUTION, HIWASSEE DAM

ticularly downstream as found at Norris Dam. The "hump" at the center is believed to be due partly to lack of uniformity in the foundation, and also to the manner in which temperature changes take place in the dam.

Attention is called to the shear diagrams. The horizontal shear diagram in Fig. 7(*a*) represents essentially temperature stresses, interpreted as being the effect of the expansion of the warm upper concrete upon the colder base. The change in the shear from that shown in Fig. 7(*a*) to that in Fig. 7(*b*) (reduced vertical scale) is roughly equivalent to the water load.

Measurements of deflection on Hiwassee and Norris dams have shown that the deflection of the top of the dam due to annual temperature variation is of the same order as that produced by water load. In certain parts of dams like Hiwassee and Norris, stresses due to temperature change are of the same order as those due to water load. When the magnitude of the temperature effects is considered, together with the indicated variation from straight-line stresses, the influence of foundation irregularity and the phenomena of plasticity and growth, one cannot rely too much on conventional calculations. Considerable skepticism is justifiable with regard to elaborate shear-twist calculations, for example.

In conclusion, there is evidence that high masonry dams actually have a greater margin of safety than is shown by the conventional calculations. So far as TVA structures are concerned, they are somewhat lighter, as a whole, than those sometimes developed by other organizations, but the evidence of the stress measurements is such as to produce confidence that the structures are entirely adequate.

*Earthquake Effects on Concrete Dams.*—One question confronting the TVA engineers was that of the possibility of earthquake effects on these structures. This was of particular importance in the lower valley. It is known that a very severe earthquake centering near New Madrid, Mo., on the Mississippi River had occurred in 1811, and there is some geological evidence that this quake had been preceded by another probably equal in intensity. The region near the confluence of the Ohio and Tennessee rivers appears to be seismically somewhat unstable and the underlying rocks in this locality are known to be much faulted. A special board of geologists reported that provision against earthquakes was advisable in dams on the lower river, but that it was not necessary for structures above Wheeler Dam. Consequently, no extra provision for earthquake effects was made in the storage dams on the tributaries or on the main river dams above Wheeler.

In the case of Kentucky Dam, however, much attention was given to this subject. Available literature and records were examined, geological advice was obtained, and some experimental work was done by Prof. Lydik S. Jacobsen at Stanford University, Stanford University, Calif., making use of a large shaking table. The main conclusions of this investigation (not published) were as follows:

1. When mass structures are founded upon rock at a reasonable distance from the center of seismic disturbance (at Kentucky Dam this may be 40 to 50 miles), the effects of the quake can be guarded against by applying, to the mass structure, a moderate horizontal acceleration. For Kentucky Dam an allowance of 5% of gravity was considered sufficient. To this acceleration effect is to be added the appropriate water-hammer effect as developed by the well-known theory of H. M. Westergaard, M. Am. Soc. C. E., if the structure is in contact with water.[3]

2. When slender structures founded upon rock are subjected to earthquake vibrations, a rational approach is to consider that a simple harmonic vibration is applied to the base of the structure and to trace the effects of this vibration throughout the structure. This includes determining the natural period of vibration of the structural element in question and then applying the assumed vibration. At Kentucky Dam this vibration was assumed to have a period of 1 sec and an amplitude of 0.25 in. (This corresponds to a maximum acceleration of 0.03 $g$.) By tracing this vibration through the structure, certain magnifications of the original impulse can be found, together with damping effects. Flexural effects on the elements will thus be determined against which suitable reinforcement must be provided.

[3] "Water Pressures on Dams During Earthquakes," by H. M. Westergaard, *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 418.

3. The investigation indicated that much greater effects were to be experienced in earth dams and in retaining walls supporting earth fills than would arise from the earthquake impulses in rock. This application seems to fall most logically in the subsequent discussion of earth dams.

## Earth Dams

Earth dams constitute a considerable proportion of the hydraulic structures of the TVA. The design of these dams has depended largely upon the use of the materials available at the respective sites. In the case of the Nottely Dam (see Table 1), suitable rock was readily available and construction conditions were adverse to a full earth dam. Consequently, this dam is a rock-fill section with a rolled core. At Chatuge Dam rock was not readily available and plenty of earth could be had, so that this dam is entirely a rolled-fill section. At Cherokee Dam the earth dam had to be located on the top of a ridge and the use of a simple earth dam would have involved excessive costs by reason of flat slopes. In this case a composite of rock and earth fill was developed.

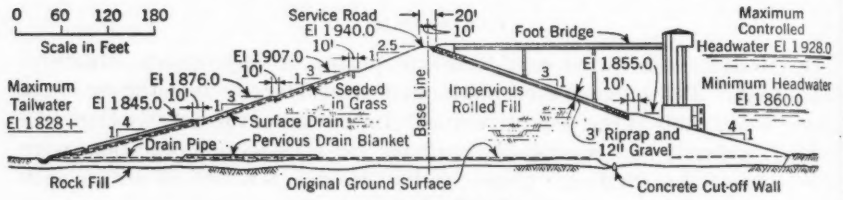
A few principles have governed the design of these fill dams. The soils laboratory determined the properties of available materials and, through field inspectors, supervised the selection and placing of the materials. The slopes of the dam sections have been generally checked by the slip circle method, using the constants determined by the soils laboratory. On the upstream face these calculations have been based on the reduced weight resulting from the sudden draw-down theory. Because this assumption is very severe, a rather moderate computed factor of safety (about 1.3) was considered sufficient. The slip circle method was applied to the downstream face also and in this case with larger factors of safety.

The primary principle enunciated in this paper applies here—namely, that the upstream face be tight and that such water as passes this diaphragm be permitted to escape readily. The minimum requirement, therefore, has included a continuous drain at the toe of the dam together with occasional wells which would drain the foundation into this toe drain. In the higher dams (Cherokee, Chatuge, and Nottely), a further drainage provision was made by actually placing a layer of porous material under a definite portion of the core or impervious element of the dam. This drainage is intended to hold down the saturation line, and to keep the downstream slope of the dam always dry, thus retaining its structural resistance. The application of these measures may be observed in the sections of Chatuge and Nottely dams (Figs. 8(*a*) and 8(*b*)).

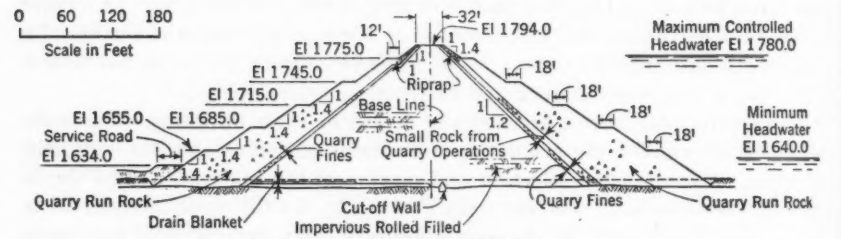
Hydraulic fill has been used on TVA dams in only one instance—the south embankment at Pickwick Landing Dam containing about 1,000,000 cu yd. In this case materials were obtained from excavation for the lock. They consisted of clay soil gradually grading down through sand into gravel. They could have been utilized for a rolled fill, but only during the dry season. A hydraulic fill can be placed in almost any weather. For this reason, the hydraulic method was followed in this instance, with satisfactory results.

At Kentucky Dam, as indicated previously, it was necessary to design against earthquakes (see Fig. 8(*c*)). The basic design for the earth section of

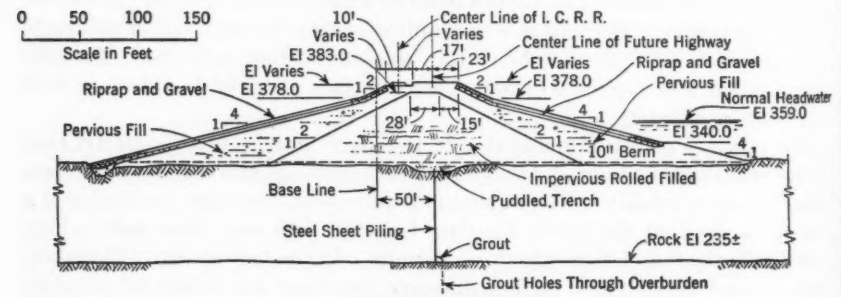
this dam, therefore, includes a core of rolled earth enclosed on both upstream and downstream slopes by additional fills of gravel and sand. The slopes (1 on 4) were built with high-friction material, and were decided to be sufficiently stable against the accelerations to be expected from the earthquake. If the




(*a*) CHATUGE PROJECT




(*b*) NOTTELY PROJECT




(*c*) KENTUCKY PROJECT

FIG. 8—TYPICAL SECTIONS, EARTH AND ROCK-FILL DAMS

rolled core should be cracked or sheared by the action of the quake, it was expected that the granular material of the shoulders would immediately fill the crack to such an extent that no material amount of water would pass.

A most important feature of design on this project is the west abutment or retaining wall which separates the spillway from the west earth dam.

The experiments of Professor Jacobsen, as well as certain experiments from Japanese sources, indicated that the center of pressure of the dynamic forces produced by the earthquake was at about two thirds of the height of the wall above the base. Furthermore, researches by the TVA staff indicated that accelerations in the ground, particularly in saturated ground, would be several times the accelerations in the foundation rock in the same quake. Accordingly, the following procedure was established for the design of retaining walls.

The horizontal acceleration in the ground is assumed at 18% of gravity. This is a judgment assumption based on examination of records, certain limitations of the Kentucky project, and the report of Professor Jacobsen. This acceleration, combined with that of gravity, is applied to the Coulomb sliding wedge and the pressure on the wall is determined. The static pressure is subtracted from this quantity and the difference is designated the dynamic increment. This increment is applied when the wall has a small batter at two thirds of the height above the base and is assumed to be the resultant of a pressure

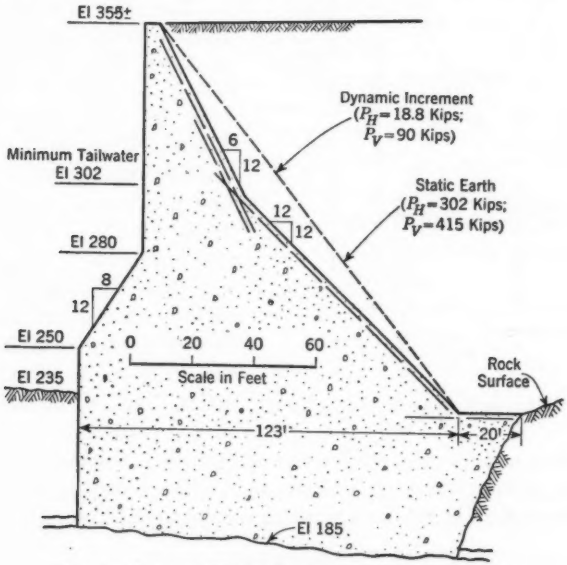



FIG. 9.—SECTION, WEST ABUTMENT OF KENTUCKY DAM

varying from zero at the base to a maximum at the top of the fill. When the wall has a large batter (flatter than 12 on 4) the assumptions as to height and distribution are a little less severe.

The application of this rule led to a maximum section on the west abutment as shown in Fig. 9. The base of this wall is extremely heavy whereas the top is heavily reinforced. The reinforced part is designed to withstand the dynamic increment at rather high stresses.

## LOCKS

Kentucky lock is the only one on the Tennessee River designed by TVA, the others having been designed by U. S. Army Engineers. The hydraulic features of this lock have been discussed elsewhere [4] but it seems worth while to explain two unusual structural features. One is the river wall of the lock, as shown in Fig. 10. The usual drainage facilities were placed in the river wall of this lock and the wall was designed with earthquake provisions. The unusual section of the river wall is explained by the fact that the layout required that the lock be built with its river wall in the reservoir, the lower pintle being substantially

[4] "Basic Hydraulics of Water Storage Projects," by George R. Rich, *Civil Engineering*, August, 1944, p. 351.

on the axis of the dam. The inner face of the lock had to be a continuous vertical wall. The outer face was obliged to carry full reservoir head with the lock empty. After investigating numerous types, the design of the double wall was adopted. The chamber in the top of the wall is filled with gravel in order to secure the benefit of its weight.

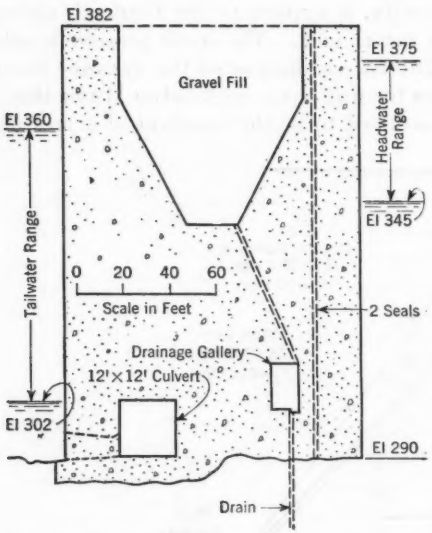



FIG. 10.—SECTION, RIVER WALL OF KENTUCKY LOCK

The other feature is a floating guide structure at the upper end of the lock, built of reinforced concrete, although at one time the plan was to build it of steel. Allowance had to be made for an unusual fluctuation in lake level; and foundation conditions, particularly in view of the earthquake risk, were very unsatisfactory for any fixed structure. Accordingly this floating guide was developed. It is 440 ft long, 24 ft wide, and is designed to be submerged about 10 ft. In its connection to the lock it is provided with a ball-and-socket traveling anchorage and its end upstream is anchored by submerged cables leading to concrete anchor blocks in the lake bottom. Aside from these provisions, the design of the lock is along conventional lines.

## SUMMARY

This paper has described some of the most important features of design of TVA structures. Every project has its peculiar hydraulic and site requirements, and its design must be based upon them. However, the paper is intended to show the lines of thought governing the solution of some major problems from the structural standpoint. No mention has been made of various special designs such as the large surge tank at Apalachia Dam,[5] the several tunnels and their appurtenances, the chute spillways at Chatuge and Nottely dams,[6] and many other interesting structures that have been described adequately elsewhere.

## ACKNOWLEDGMENT

The three papers of this Symposium represent activities of the design organization of the TVA. All engineering and construction work of the TVA is under the direction of C. E. Blee, M. Am. Soc. C. E., Chief Engineer. The design of structures and accessories, as well as the specification and inspection of equipment, has been the responsibility of the Design Department under the direction of George R. Rich, M. Am. Soc. C. E. until recently Chief Design Engineer.

[5] *Civil Engineering*, October, 1943, p. 465.
[6] *Ibid.*, p. 413.

# DESIGN OF HYDROELECTRIC STATIONS

By George P. Palo,[7] and Constant R. Marks, III,[8]
Assoc. Members, Am. Soc. C. E.

## Synopsis

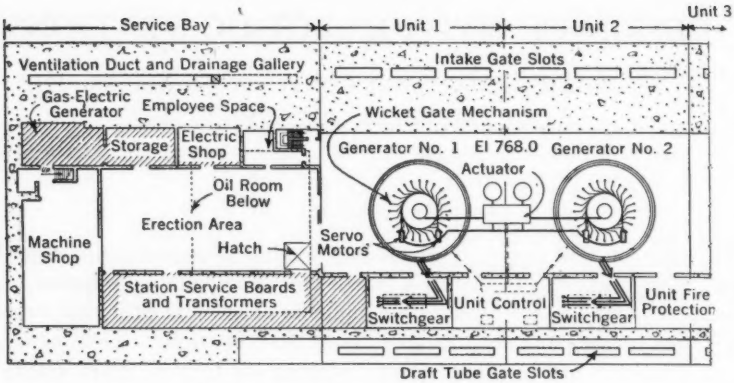
The title of this paper, if given its broadest interpretation, describes a subject of such scope as to require a treatise for its adequate discussion. Actually, this scope is limited to certain comments on general problems connected with station layouts and substructure design for large units. In discussing the layouts, emphasis will be placed upon designing the form of the structure most efficiently for the functions it must serve and upon the effect on the design of the interdependence of many of the factors involved. Illustrations will be drawn from the projects of the Tennessee Valley Authority (TVA), which include fourteen powerhouses and additions to one, for a total of forty-two units with a capacity of about a million and a half kilowatts. Discussion will be directed particularly to low-head, propeller type installations of which TVA has built seven, six with Kaplan runners. The locations of TVA projects are shown on the map in Table 1.

## The Functional Layout

Securing a satisfactory and economical layout for the powerhouse should be one of the first concerns of the designer. A study to create a properly functional layout begins from the conception of a hydroelectric power station as a mechanism for converting the potential energy of water under head into electrical energy. The various features of the powerhouse may then be arranged in a manner suited to the several component functions necessary for accomplishment of this primary function.

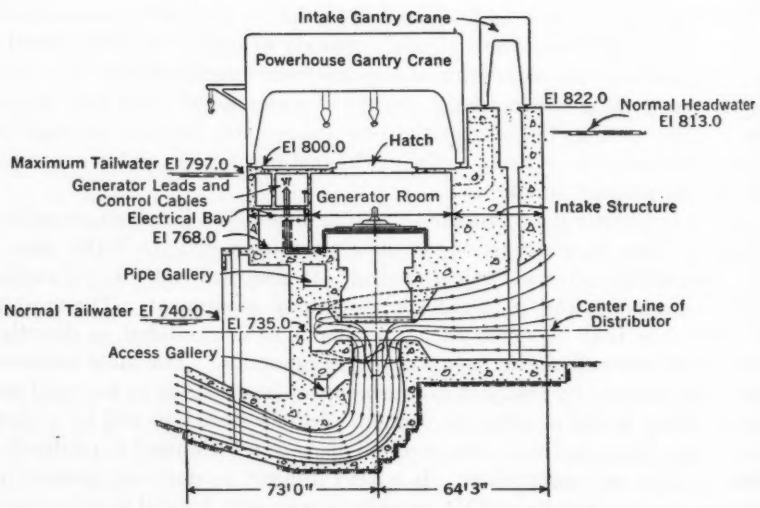
Fig. 11 is a partly diagrammetric drawing of the Fort Loudoun powerhouse, planned to show its six functions and their arrangement: (1) Water flow; (2) flow of electricity; (3) mechanical control; (4) electrical control; (5) auxiliary electric equipment; and (6) auxiliary mechanical equipment. The most efficient layout is that in which each function is accommodated as directly as possible and without interference with other functions. The most economical structure is secured by compact arrangement of functions so as to avoid waste space in either initial or ultimate portions of the installation and by a plan in which as much as possible of the required space can be housed in relatively inexpensive types of construction. It is very difficult to state any general rules for layout, but the results of TVA experience may offer helpful hints for similar installations.

[7] Lt. (jg), U.S.N.R., Alexandria, Va.
[8] Senior Civ. Engr., TVA, Knoxville, Tenn.



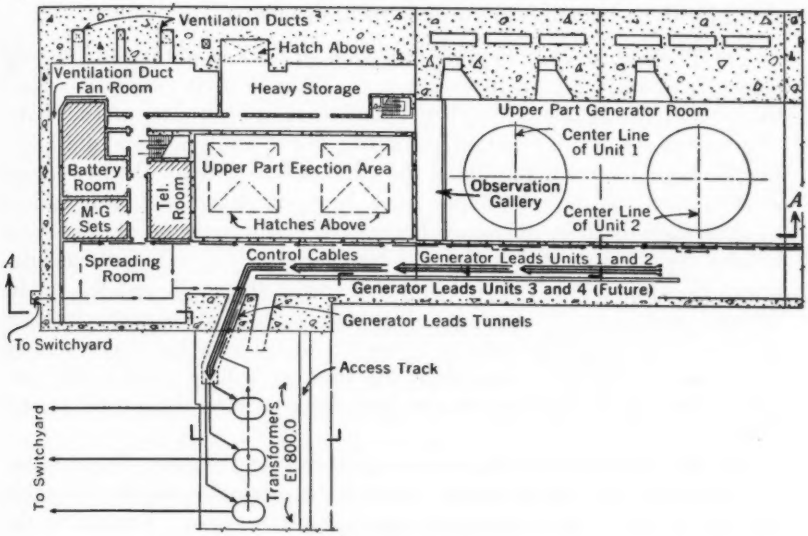

(*a*) PLAN OF GENERATOR ROOM FLOOR

| LEGEND | | |
|---|---|---|
| NO. | FUNCTION | SYMBOL |
| 1 | Water Passages | |
| 2 | Electric Current | |
| 3 | Mechanical Control | |
| 4 | Electrical Control | |
| 5 | Auxilary Electrical Equipment Space | |
| 6 | Auxilary Mechanical Equipment and Space for Employees and Visitors | |

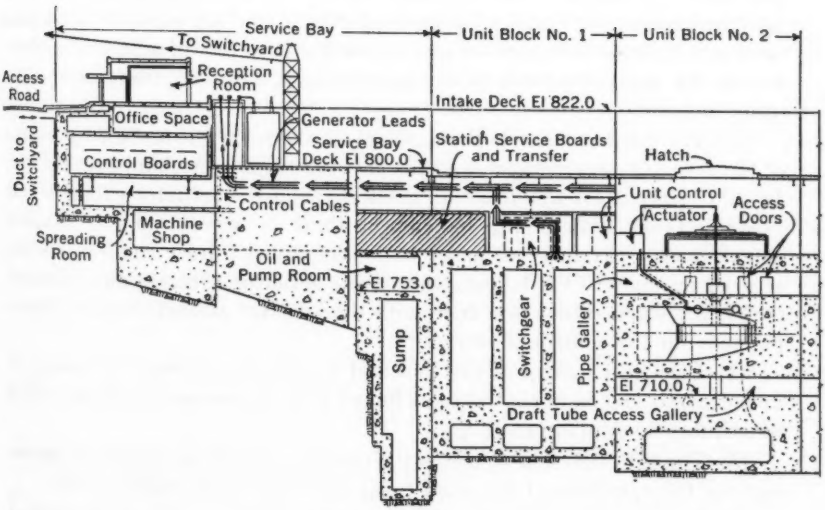



(*b*) TYPICAL SECTION ON CENTER LINE OF UNIT

FIG. 11.—FORT LOUDOUN POWER-




(c) PLAN AT EL 783.50




(d) SECTION *A-A*

Consider these functions in greater detail, as follows:

(1) The first function requires provision for the flow of the water through passages, where it imparts its energy to the turbine runner, and protection of the powerhouse against all expected conditions of headwater and tailwater. The waterways of this function, consisting of the scroll case and draft tube, are the basic factors determining the minimum outline for the powerhouse substructure. Waterway shapes have been simplified and standardized so that there is little difference between the requirements of the several turbine manufacturers. In fact, the designer may cooperate with manufacturers to obtain a single design in which any of them will install a turbine without qualifying his guarantee of its efficiency. This is particularly desirable if it is intended to complete, partly, the substructure to be used for future units. Further comment on this function will be made in the latter half of this paper, in which substructure design will be considered at some length.

(2) The second function involves provision for the flow of electric current, starting from the generator, through the conductors to the transformers, and thence to the switchyard and transmission lines. Layout for this function involves a consideration of the location of transformers on the intake deck, on the powerhouse roof, over the draft tube, or, at a greater distance from the generators, on the river bank near the powerhouse. Decision as to the best location depends on many factors, but in planning a layout the space requirement for the function should be understood clearly. In TVA practice, for example, transformers are usually on the bank, and low-tension electrical equipment is at a minimum. In the layout the engineer must plan for space for low-tension switchgear for each unit or pair of units and a clear run of the generator leads to the transformers, without interference with other features.

(3) The third function is the mechanical control of the turbine by means of the actuator, servomotors, and the gates or Kaplan mechanisms (see Fig. 11(*a*)). Layout for this function must envision the actuator cabinets and pressure tanks with piping to connect them to servomotors or Kaplan mechanisms. Economy of space and economy of direct connections are secured on many TVA layouts by placing actuator cabinets and pressure tanks on the generator-room floor, with the actuator cabinet planned, where possible, for the control of two units.

(4) Electrical control of the unit and the flowing current by means of unit controls and main control room linked with the necessary electric cables constitutes the fourth function.

(5) The fifth function concerns the auxiliary electrical equipment necessary for the operation of the main equipment and of the station itself.

(6) Finally, the sixth function constitutes the auxiliary mechanical facilities required for servicing the equipment, provision for access into and within the station, and space for use and convenience of employees and visitors.

The arrangement of the facilities serving functions (4) to (6) may vary greatly, depending on the number and size of the units, topography, type of superstructure, and other conditions. Design for economy and simplicity of layout can be aided by reclassifying these facilities, and the second and third functions as well, into unit service features and station service features. Unit service features include the mechanical control of the unit, the unit portion of the electrical control, and the provisions for physical access to the unit areas and access of leads, control cables, and pipes. Station service features include the main control area and nearly all of the auxiliary mechanical and electrical equipment as well as the employee and public space. The unit service areas are repetitive for each unit. The station service features, on the other hand, need be provided only once for the entire station. If the number of units is large and the cross section of a unit block is to be kept typical, structural economy can be secured only by limiting the space in the unit block to the minimum required for unit service features. This is particularly true if provision for future units with the same cross section is being made; for, in this case, station features must all be included in the initial construction and only additional unit features are needed when the structure for the future units is completed. Economy generally demands that station features be located in one or more service buildings separate from the unit blocks. An exception is an installation of from one to about three units where shortness of connections and operating convenience may favor location of station service features in a more central location.

The Fort Loudoun cross section is typical for several of the low-head stations of the TVA. The unit service area uses a portion of the generator-room floor for the actuator, a two-story electrical bay (one for unit control and switchgear, and the other for clear runs of generator leads and control cables), and galleries in the heavy concrete sections above and below the scroll case. Recesses from these galleries between units provide space for auxiliary mechanical unit service features which is conveniently segregated from other features. The resulting structure is a very compact, economical cross section.

The station service area is most commonly housed in a service bay at one end of the series of unit blocks. This structure usually provides an erection area at the level of the generator-room floor. As shown for Fort Loudoun, a machine shop at the same level, and space on a lower floor for station sump pumps and lubricating oil servicing, are desirable and may be planned readily in conjunction with the mechanical equipment at the same level in the galleries and recesses in the unit blocks. Other station service features should be located within the service bay so as to serve best the function of each feature and to fit them into an economical layout which coordinates with the main functions of the powerhouse. Space provided may be affected by the shape of service bay best suited to the structural problem of its design. If the structure is located in the river channel, it must be massive enough to be stable against high tailwater levels and uplift, against headwater if it forms a part of the dam, and possibly as a retaining wall if it is the transition between concrete and earth sections of the dam. If the river bank consists of suitable material for foundations and dam abutment, some of this massive structure may be saved by locating the

service bay so close to the bank that a part of it may be founded at elevations well above the river bed, as was done at Fort Loudoun. Likewise, if space in the river channel is limited or if structures within it are expensive due to possibility of high tailwater, large savings may be made by a layout in which all station service facilities possible are moved to a separate structure located on the bank above maximum tailwater level. The Watts Bar project of the TVA is an example of such practice.

## The Superstructure

The planning of the powerhouse layout requires early decision as to the use of indoor, outdoor, or semi-outdoor type of superstructure. A consideration of factors governing this choice will illustrate the interdependence of some of the several functions thus far discussed.

For maximum tailwater below the level of the generator-room floor, only indoor and outdoor types usually need be considered. Controlling factors are: Relative economy; operating convenience (involving alternate solutions to the

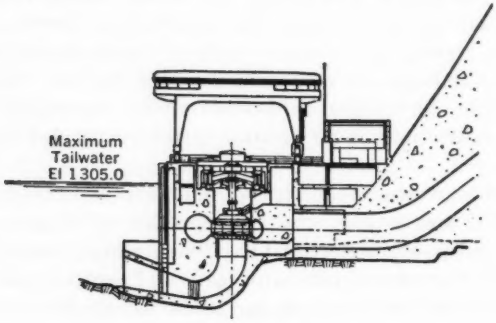


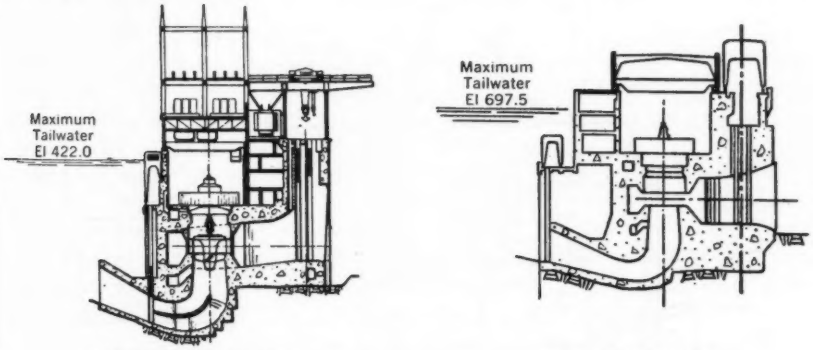

Fig. 12.—Hiwassee Powerhouse

problem of the mechanical service function); and appearance. There are many examples of each type of design, indicating the difficulty of evaluating these factors, and the choice of type must be considered largely a matter of organization preference and policy.

As larger streams with flatter gradients are developed and as flood studies and experience with actual developments point toward more conservative choice of maximum flood, many projects must provide for tailwater well above the generator-room floor. In this case layout requirements, particularly the one of access to the powerhouse, become of great importance in determining the proper type of superstructure.

For maximum tailwater a moderate distance above the generator-room floor level, the full outdoor type of design may be modified by building water-bearing walls to a height approximating that of the generator barrel and by roofing over the generator area. The resulting section may be like that for the Hiwassee project (Fig. 12). The space enclosed around the generator is useful, especially for electric services, and this type of outdoor station has sometimes been used for that reason alone. For high tailwater, access is at the level of the roof.

The indoor powerhouse can be adapted with minor modifications for tailwater a short distance above the generator-room floor in several ways. One solution is to locate the main entrance on a mezzanine which is above tailwater. Both size and elevation of this floor must be considered with care to minimize waste space either on or below the floor. Another solution, preferably only for small stations, may locate the entrance to the powerhouse at the generator-room floor level and reach it by a protected ramp which slopes from an entrance area above high tailwater; or, in rare circumstances, it may be considered reasonable to protect the powerhouse entrance with a temporary bulkhead set in place during floods. For conditions of tailwater more than about 15 ft above the generator floor, these solutions are rarely satisfactory. The indoor type of powerhouse can be adapted economically to very high tailwater levels only by providing access through a vertical hatch. In the Pickwick project (Fig. 13(*a*)),




(*a*) PICKWICK LANDING (*b*) CHICKAMAUGA

FIG. 13.—INDOOR POWERHOUSES

delivery of equipment is by barge and a crane transfers it from the barge to a hatch in the roof of the powerhouse immediately above the erection area. At the Chickamauga project (Fig. 13(*b*)), there is a railroad or truck access to the intake deck. The gantry crane used for moving intake gates can also handle incoming equipment and drop it through a hatch to the level of the generator-room floor. At this level it is moved downstream on a transfer car until within reach of the crane in the generator room. It will be noted that this layout creates waste space at the hatch opening and also prevents, for some 45 ft above the generator-room floor, any straight run of control cables or generator leads upstream from the generators through the length of the powerhouse. This is one of the factors requiring that the electrical bay for the project be on the downstream side of the generator room. At Pickwick, on the other hand, there were a number of factors pointing to an upstream electrical bay, and the need for a cable gallery necessitated the opening in the roof.

The TVA has found the semi-outdoor type of powerhouse well adapted to many conditions. In some projects it has been the only reasonable solution. As illustrated by the powerhouse at Kentucky Dam (Fig. 14), the generator room is about 30 ft high, ample to clear the Kaplan head on the unit. The ad-

jacent electrical bay is two floors high and the downstream and end walls provide protection against tailwater 28 ft above the generator-room floor. A gantry crane hoists equipment through hatches in the roof, and the main access road comes in at roof level. If the Kentucky section is compared to that for Hiwassee, it will be noted that the main difference is that the roof at Kentucky is at sufficient height to justify the design of a generator room, whereas at Hiwassee the lesser height leads to the adoption of a concrete barrel around the generator.

The TVA has used the semi-outdoor powerhouse at five projects and this design has proved to interfere less with other functions and to be more satisfactory than any other. At four of the projects (two Kaplan and two Francis

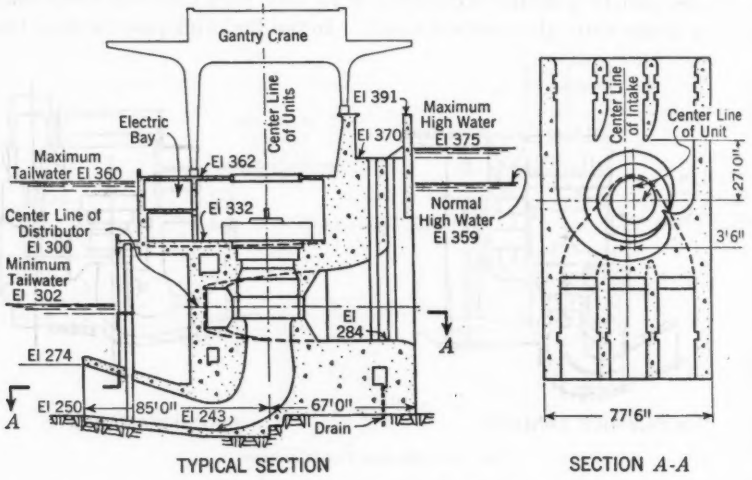



FIG. 14.—SEMI-OUTDOOR POWERHOUSE

wheel installations), tailwater may be 20 ft to 30 ft above the generator-room floor. At Watts Bar (the first project to use this design), maximum tailwater is only 13.5 ft above the generator-room floor. Here, however, the service bay is built against a precipitous abutment which precluded use of an end entrance. To have used a downstream entrance to a mezzanine would have interrupted cable runs in the downstream electrical bay which were necessary to permit use of what was considered the most economical cross section of intake and powerhouse.

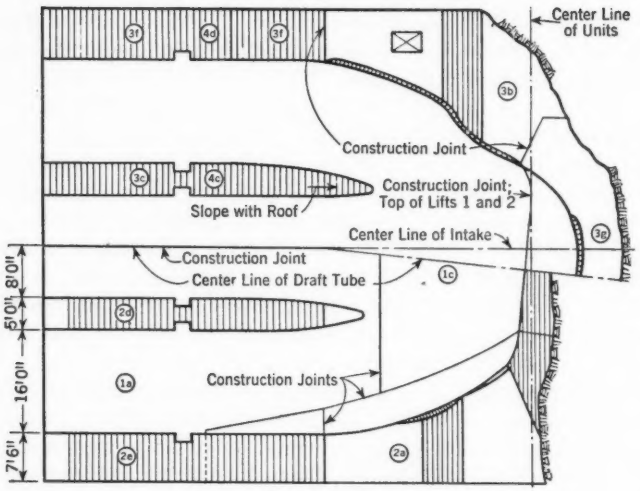
## SUBSTRUCTURES FOR LARGE PROPELLER UNITS

The shape of the substructure required for large propeller units prohibits calculation of anything more than a crude approximation of the stresses involved. Even structures with identical outline and identical water loads will have different stresses, depending upon the sequence, size, and shape of the concrete pours, the conditions of temperature encountered during the construction, and the properties of the concrete used. Indeed these factors may account for

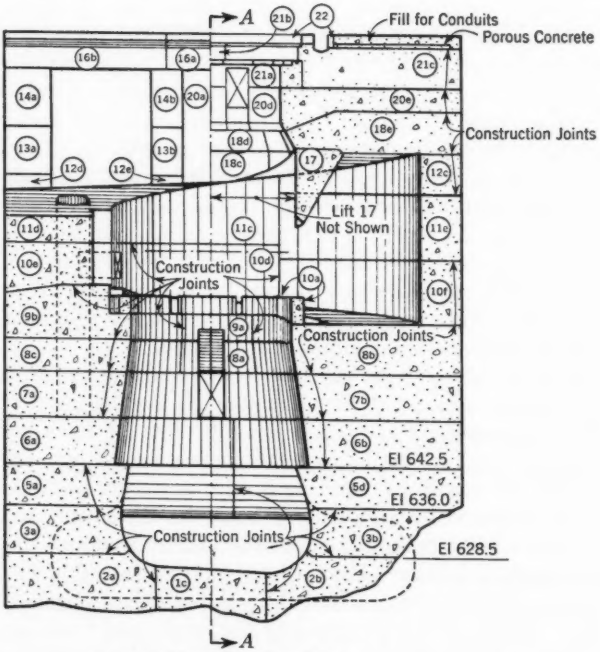
the major portion of the stress in the structure. The objectives of the structural design are (*a*) to obtain an economical structure safe from actual failure, and (*b*) to make the structure watertight. The fact that actual failures are almost unknown indicates universal success in the achievement of the safety, but not necessarily the economy, of objective (*a*). That objective (*b*) is obtained less frequently is evidenced by casual inspection of many power plants. The benefits of a watertight structure are: More satisfactory operating conditions; less depreciation and maintenance; and assurance of greater over-all strength in the structure. In addition, watertightness indicates reduction of movements in the structure, so that there is more certainty that the turbine and generator equipment will be erected and remain more closely to perfect alinement. These benefits are deemed to warrant the extra care and expense in the design and construction needed to obtain them.

*Safety and Economy.*—The cross section of the Fort Loudoun powerhouse is an example of structural safety, with economy, obtained by designing the intake and powerhouse to function as a unit. Two units were completed initially but the success of the section shown in Fig. 11 was dependent on its adaptation to initial construction for two future units. For these units the section is complete except for most of the scroll-case roof, the lower cone, and a recess sufficient to permit the placing of the draft-tube liner. Such construction requires extensive commitments relative to waterways in these future units. Equal commitments in plants built in the 1920's were found unsatisfactory within a few years because of development of the propeller wheel and the use of the elbow draft tube. There have been no major changes in turbines and waterway design since before 1935 and those commitments may now be made more freely for future units to be installed in a reasonable time. This is a fortunate condition because, in many large propeller installations, foundation conditions require a design that will buttress the inadequacy of the intake foundation when the draft tube is excavated to a greater depth than that required for the intake. This buttressing is obtained by completing a large part of future units. In addition, partial construction of future units is almost always included to permit unwatering the area of future construction by dropping stop logs or gates into slots in the partly completed draft tube.

*The Roof of the Intake.*—Perhaps the most critical design feature in a cross section for a future unit, as at Fort Loudoun, is the design of the roof of the intake. Water passages of the intake are protected by trashracks set against the upstream face and controlled by service gates moved by a gantry crane on the deck. For rare occasions requiring closure upstream from the service gate, trashracks are removed by an auxiliary boom on the gantry and replaced by stop logs. This simple method of control is a factor that aids the development of a short, economical intake. For the head on this project, however, the intake is not stable by itself and omission of the scroll-case roof requires that much of the reservoir water load be transferred by the intake roof from the intermediate to the end piers, which, with the scroll-case walls and draft tube, have the length necessary for stability of the structure. The framing of the intake roof with the end piers should be such as to insure spreading of the load so that the full length of the draft tube is actually effective as a gravity section. At Fort

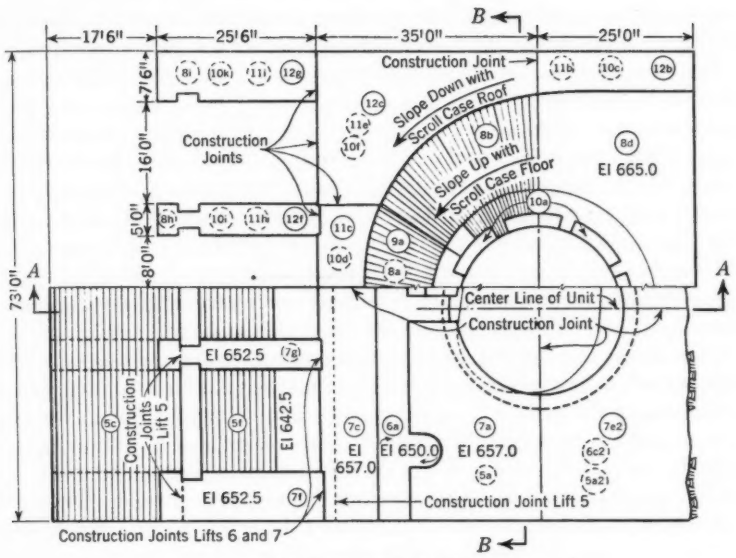



(*a*) HALF PLAN, LIFTS 1 TO 4

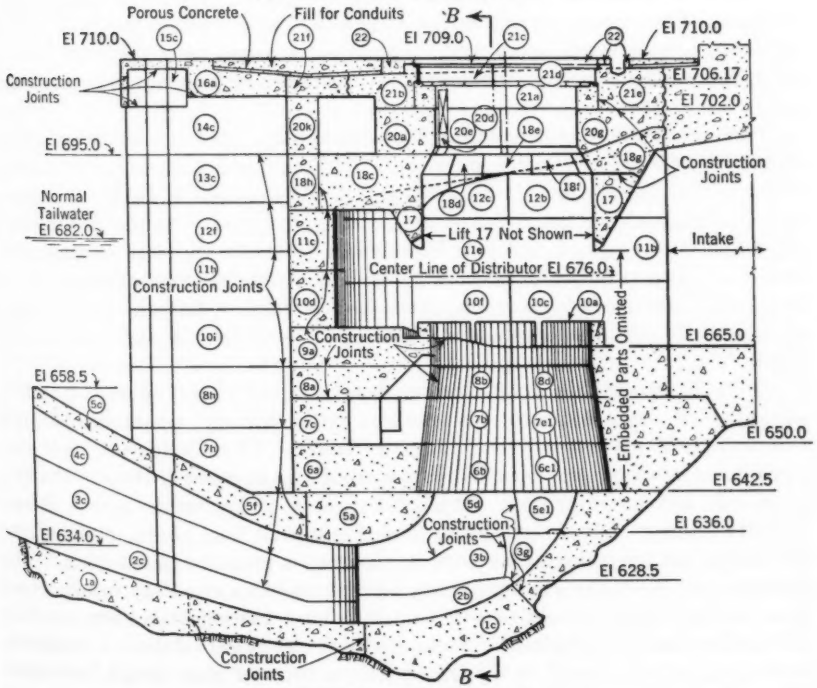



(*b*) SECTIONAL ELEVATION *B-B*

FIG. 15.—LAYOUT OF CONSTRUCTION




(c) HALF PLAN, LIFTS 5 TO 12




(d) SECTIONAL ELEVATION *A-A*

JOINTS, WATTS BAR POWERHOUSE

Loudoun this was accomplished by placing, initially, a part of the scroll-case roof, upstream from the cone, to function as a haunch on the intake roof beam.

*Watertight Structure.*—Throughout the design, the civil engineer must constantly consider the problems of volume change if he wishes to obtain a waterproof structure. Volume change as applied to a powerhouse substructure (most of which will be permanently submerged) arises largely from temperature changes in the concrete—initially as a result of the heat of hydration of the cement and later as a result of the annual temperature cycle. The magnitude of the change, and hence of its accompanying problem, will depend in part on the season of construction and the thermal coefficient of the concrete. These factors indicate the individuality of projects; yet certain general approaches to design are desirable even under most favorable conditions.

A problem encountered early in construction is the spacing of contraction joints in the powerhouse substructure. Units spaced 35 ft to 40 ft or more on centers should be built with contraction joints between units. Smaller unit spacings may be built with two units between pairs of contraction joints. For simplicity and clean design, in both substructure and superstructure, it is desirable that contraction joints be straight. The unit is always offset toward the small side of the scroll case and, if the draft tube is made symmetrical about the center line of unit, the end piers of the draft tube will not be of equal thickness. This lack of symmetry is undesirable in a structure planned for temperature changes. Best results are obtained if the draft tube is offset in the heavy elbow section so as to bring the center line of the downstream end of the draft tube on the same center line as that for the intake, and midway between the contraction joints.

With contraction joints established, consideration may turn to the individual unit. Fig. 15 affords some conception of the planning of the construction joints in the Watts Bar Hydro Station. Units are spaced 73 ft on centers, and the length from the construction joint at the intake to the downstream end of the draft tube is 100 ft. The design considers concrete outline, reinforcement, construction joints, and special concrete pouring sequence as an integrated problem with the objective of a strong, watertight substructure which can be obtained with a minimum of restriction on the construction forces.

*Draft Tube.*—In the draft tube the length of individual pours is held to a maximum of about 40 ft by construction joints near the center line of units and near the downstream wall of the powerhouse. These joints are made with a minimum of offsets at successive lifts so that any opening of the construction joint will not continue into monolithic concrete in another pour. Where stability requires that shears be transferred across vertical joints, joint surfaces are roughened by jackhammers before concrete is poured against them. The downstream end of the draft tube may be designed as a one-story frame. End piers for large units should be 6 ft to 8 ft thick. Intermediate piers must be 5 ft to 6 ft thick to provide for reasonable thickness at gate slots. As temperature change is considered in the frame design, the roof slab should be as light as consistent with reasonable reinforcing requirements. A thickness of 4 ft is recommended. One or two construction joints in its width also minimize

temperature stress. Horizontal steel in the draft-tube piers is designed as 1¼-in. square bars, 12 in. on centers, so as to spread the cracks that will result as the concrete changes length over the restraining rock foundation.

The key to successful construction of the massive elbow of the draft tube is found in the proper concreting of the roof pours. In the entire substructure a time interval of three days between successive vertical lifts is specified; but, where effects of temperature are so critical that more complete dissipation of the heat of hydration is essential, longer intervals between successive lifts or adjacent pours are required. The elbow zone is particularly important as cracks will cause leaks into the draft-tube gallery with eventual large accumulations of laitance. The roof pour is made with one or two construction joints in the width of the unit, and a delay of seven days is specified between the placing of adjacent pours and between the first and second lifts. This gives the concrete time to dissipate a part of its heat of hydration and also to develop strength to carry the weight of succeeding pours. Roof reinforcement of 1¼-in. square bars 9 in. on center has proved satisfactory.

Additional features of interest in the design of the draft tube are the slope of the floor and the outline of the upstream nose of the intermediate piers. As the low point of the draft-tube excavation is usually well below the level of rock in the river bed, a steeply sloping draft-tube floor is usually desirable to reduce rock excavation not only in the draft tube but also in the tailrace channel. On TVA projects, floor slopes of 1 on 3 or even 1 on 2.5 have provided perfectly satisfactory tubes. Since most of the energy remaining in the flowing water is lost on emergence from the draft tube, it is necessary only to have the draft-tube outlet low enough to seal during the operation of the unit.

For hydraulic reasons manufacturers often request that the upstream end of the intermediate piers be rounded to a small radius and located as far downstream as possible. Structurally, these points are obviously subjected to considerable stress. On the Guntersville project an installation of Carlson strain and stress meters indicated stresses in the pier noses of about 400 lb per sq in. Stress analysis which considered concrete placing sequence and length change of piers under load had previously indicated this to be the probable stress although higher values had been considered possible. To allow for approximations in these data, pier noses on TVA projects are located about as shown for Watts Bar. They are rounded to a 6-in. radius, which is a good radius hydraulically, but also large enough to permit embedment of substantial, tied-column type of reinforcement.

*Scroll Case.*—The scroll case is usually built in two stages, the first of which permits the superstructure erection before the delivery of the parts of the turbine that are to be embedded. Such parts can then be handled by the powerhouse crane. The first stage consists of a U-shaped section formed by the downstream and side walls of the scroll-case passage. The downstream wall is carried to full height to provide support for the superstructure construction. The side walls may be carried to full height or may stop at the top of the scroll-case waterway. The design of the side walls of the scroll case requires that they be strong enough to carry the final water loads by beam action between the floor and the roof of the scroll case and that they be capable

of taking certain amounts of stress caused by temperature change in the roof concrete. If a wall is increased in thickness to reduce the reinforcing for water load, its temperature stresses are increased. Accordingly, there is an optimum wall thickness that best meets the requirements of both water load and temperature. In scroll cases approximately 30 ft high with water pressures from heads of 50 ft to 70 ft, this thickness is about 6 ft or 7 ft. This thickness is also suitable for carrying the shear stresses without special reinforcement. The scroll-case walls must also be designed for any high tailwater conditions to which they may be subjected before the scroll-case roof is in place.

Scroll-case walls are subject to considerable horizontal stress from temperature change. For example, at Pickwick, Chickamauga, and Guntersville, a vertical crack occurred in approximately the bottom third of the scroll-case walls about 10 ft upstream from the center line of the unit. At other projects it has been possible to observe considerable similarity in crack patterns among

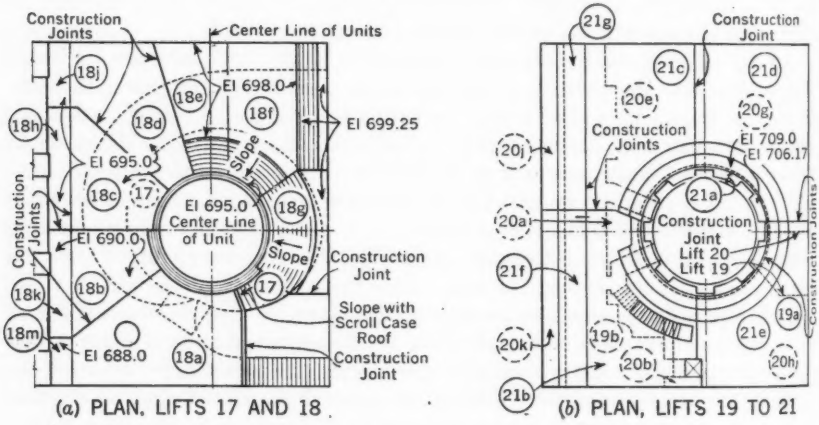



FIG. 16.—SCROLL-CASE CONSTRUCTION JOINTS, WATTS BAR POWERHOUSE (SEE FIG. 15)

several different units, indicating that these cracks are caused by a definite set of conditions which is repeated from unit to unit rather than by any chance concentration of stress. The most satisfactory method for placing the first stage of the scroll-case walls (see Fig. 16) has been found to be to pour them in five sections. Three, consisting of the central portion of the downstream wall and the upstream half of the end walls, are poured first. Then, after approximately one week has elapsed, the heavy corner sections are placed. Where future units are being constructed, this procedure may be improved by leaving a slot approximately 5 ft wide between intake pours and the scroll-case pours. This slot is then filled with concrete in cold weather.

The major portion of the second-stage concrete is the roof of the scroll case. This concrete must be reinforced heavily with radial bars to anchor the shears and moments carried to it from the scroll-case walls. However, it too is subjected to very high temperature stress. For example, at the Pickwick project, where one half of the scroll-case roof was poured at a time, the major

stresses, before the scroll case was filled, were in a circumferential direction as indicated by the development of a system of radial cracks. To reduce these stresses, the first lift of the roof is placed in eight pie-shaped pours. Four, in opposite pairs, are poured simultaneously, one week after completing the upper cone. After one more week, the four intervening pours are made simultaneously. Successive lifts are in four pieces. This procedure, in addition to reducing temperature stress, also permits more accurate erection of the unit. Before its adoption, the pouring of the first part of the scroll-case roof was immediately followed by shifting of the speed ring, as the newly poured concrete first generated heat and then dissipated it. The placing of the opposing pie-shaped pours greatly reduces this movement. As a final precaution to guarantee against difficulties due to possible leakage through the scroll case, a layer of porous concrete, drained to the powerhouse sump, should be placed on the generator-room floor below the finished surface.

## Conclusion

A few of the problems encountered in the design of hydroelectric power stations have been discussed briefly in this paper. In the rapid development of this field of civil engineering, much of it has received inadequate treatment in books or papers, probably because of the realization that such papers would become obsolete in a short time. Development is not at an end, of course, but it is believed that present knowledge of powerhouse design is broad enough and that the pace of its development is sufficiently slowed so that an extensive discussion of the subject, widely distributed among designers, could be used to obtain better designs. Should this paper help to achieve this end, it will have accomplished a fundamental purpose.

# CIVIL ENGINEERING FEATURES OF STEAM PLANT AT WATTS BAR

BY ADOLF A. MEYER,[9] M. AM. SOC. C. E.

SYNOPSIS

The civil design features of the Watts Bar Steam Station are described in this paper. The general station design and its functions within the system defined as the Tennessee Valley Authority (TVA) have been described elsewhere.[10] Only references to the general design necessary for the description of the civil engineering features are included herein.

The paper gives the reasons why the station was located on the high ground about 600 ft away from the river rather than at the river front, as is the more common solution. It outlines the characteristics of the foundation rock—a shale of very low structural strength—and the precautions used in the design of the substructure to avoid detrimental settlements of the building and to eliminate the formation of shrinkage cracks.

The paper explains the conditions which led to the adoption of a gravity feed system for the condensing water supply, and it describes the main features and the functions of this system. It further describes the planning and design of the powerhouse structures, the yards for coal handling and coal storage, and the transportation facilities connecting these yards.

The Watts Bar Steam Station was authorized on July 31, 1940, shortly after the fall of France. It was to increase the power capacity of the TVA system to insure adequate supply for the armament program then under consideration. The initial authorization was for a capacity of 120,000 kw. The ultimate station installation was to be 240,000 kw. The load center for the armament program was assumed to be in east Tennessee. As fuel costs in this area are favorable, the logical site for the new steam station was in this section at a location near a primary substation with ample transmission interconnections, good transportation facilities, and an adequate cooling water supply. These conditions were best met near the site of the Watts Bar hydroelectric development, on the Tennessee River halfway between Knoxville and Chattanooga, Tenn. At that time (1940) this project was in the course of construction. By combining the two projects, substantial advantages could be obtained from savings in construction plant and in construction supervision, in addition to the permanent operating advantages.

The paper will be limited to a treatment of the civil engineering features of the station. No sharp separation of these elements from the integrated plant can be made. However, civil engineering design predominates in the

[9] Head Civ. Design Engr., TVA, Knoxville, Tenn.

[10] "The Watts Bar Steam Power Station of the TVA," by G. R. Rich and R. T. Mathews, *Mechanical Engineering*, November, 1941, p. 773.

following features: Powerhouse substructure, powerhouse superstructure, condensing water supply, yards for coal storage and coal handling, railroad facilities, road facilities, and river dock facilities.

At Watts Bar, the combined cost of these items represented approximately 20% of the total cost of the station. On first appearance it may be concluded that they did not affect the economy of the station to a marked degree. This impression, however, would be misleading. The largest items of cost are incurred in the main equipment, the purchase price of which is not very flexible for given specifications. Therefore, the cost increment involved by the civil engineering features is the most variable part when comparing different stations. For these reasons, it was considered a matter of great importance for the over-all economy of the station that a careful evaluation of the civil engineering features be the guiding rule in the selection of the site.

The Watts Bar Dam site is characterized by a steep bluff, approximately 150 ft high, on the right bank of the river. A railroad is accessible from this side only, which eliminated from consideration the wide flood plain on the left bank of the river. Sites upstream and downstream of the dam on the right bank were investigated. The costs at the upstream sites were high because an excessive amount of rock excavation was required. Approximately one-half mile downstream from the dam where the river bluff recedes, very favorable conditions for foundations and yard grading could be found. There the rock underlying the flood plain gradually rises away from the river. After a study of requirements for condensing water circulation and of the protective measures against water pressures during flood stages had established the optimum elevation of the building foundation, the specific site of the powerhouse could be chosen at a place where firm rock was at this optimum elevation. Railroad and highway facilities and a wharf area at the river front were all in immediate proximity to this area.

The generating station was built to house four units of 60,000 kw each. The plant is designed upon the unit principle; that is, all auxiliary and control equipment is arranged to allow each boiler-turbo-generator unit to operate as a self-contained plant (see Fig. 17).[11] At each such unit, coal is delivered from a bunker to four pulverizers which connect to the eight burners at the boiler. The boiler has a rated capacity of 600,000 lb of steam per hour. Steam, with a pressure of 900 lb per sq in. and a temperature of 900° F, is furnished to the 17-stage, single-cylinder turbine, which operates at a speed of 1,800 rpm. Current is generated at 13,800 v, which is stepped up to 154,000 v at an outdoor transformer yard adjacent to the turbine room. Because of the urgency of early completion of the station, all equipment was selected on the basis of established proven merit, with departures into new fields of design purposely ruled out.

The framing layout of the steel superstructure was developed hand-in-hand with the early equipment arrangements. The housing enclosure was to be kept to a minimum size and was to be built of members easily procurable, provided that these objectives did not prevent obtaining an accessible arrangement of all equipment.

[11] "The Watts Bar Steam Power Station of the TVA," by G. R. Rich and R. T. Mathews, *Mechanical Engineering*, November, 1941, Fig. 4, p. 776.

No contraction joints were used in the framing for the 250-ft building. Bracing was introduced close to the sources of horizontal loads. The end bays, where wind and crane bumper loads act, are braced.

For reasons of economy and dependability of service, the coal bunkers are designed to contain a supply of coal sufficient for 1.5 days. They are large

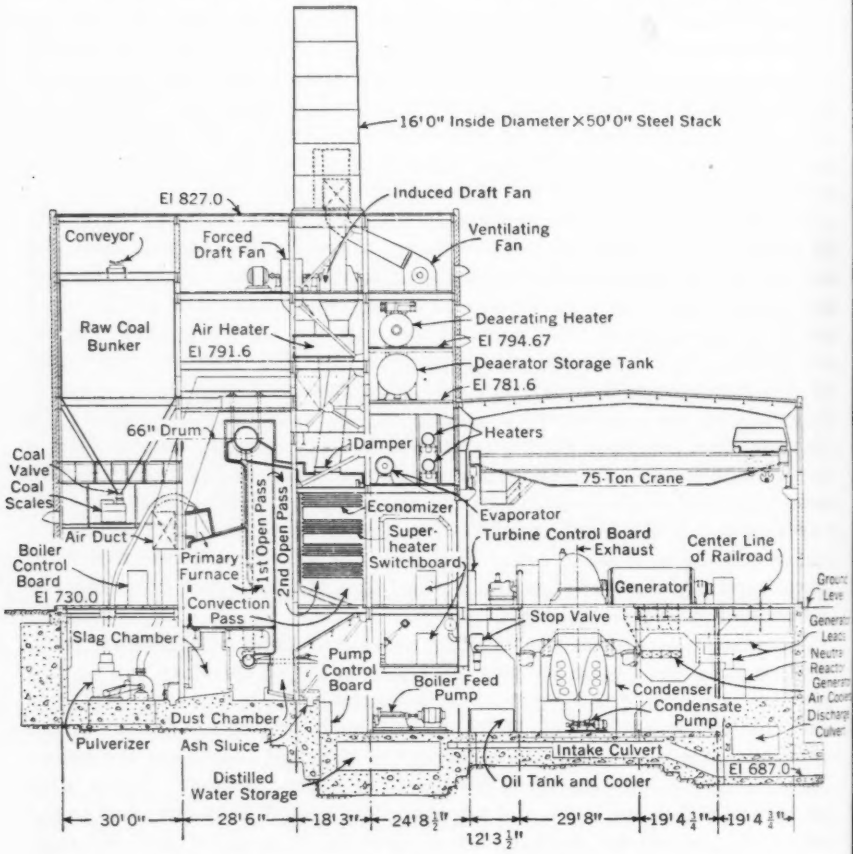



FIG. 17.—CROSS SECTION SHOWING ARRANGEMENT OF EQUIPMENT

structures of 1,250-ton capacity each. The plates in the lower part are of special abrasion-resistant steel. All other bin plates are of copper-bearing steel to retard corrosion.

The heavy boiler loads are suspended from heavy girders approximately 50 ft above the generator floor. For the generator-room framing, a welded frame fabricated from 33-in. wide-flange (WF) beams was used. Because this type of construction saved approximately 7 ft of building height, its cost compared favorably with the conventional truss layout. In appearance it is superior to any type of truss. The detail of the crane bracket itself is worth men-

tioning. A section of the column web, 3 ft long, was removed and the inside column flange was slotted. The 3-in.-thick bracket plate was then inserted and welded on all sides.

Because early delivery of the fabricated steel was essential, built-up sections and special shapes were generally avoided. Continuity of the framing helped a good deal in accomplishing this goal. The total weight of the steel framing of the completed four-unit powerhouse amounted to 4,200 tons.

The station location could be chosen at a place where firm rock was just slightly above the elevation of the boiler foundation. For the low part of the foundation under the condenser pit, the rock was removed easily (see Fig. 18).

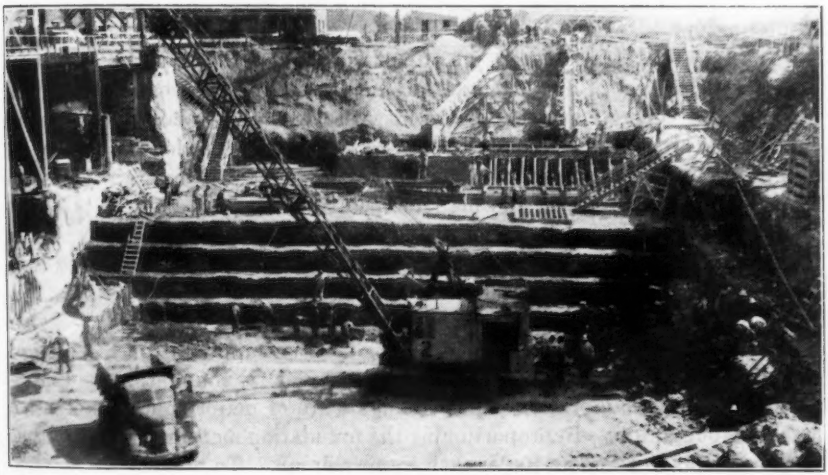


FIG. 18.—EXCAVATION FOR THE POWERHOUSE

However, this very favorable condition had one serious drawback. The structural strength of the foundation rock—a sandy fissile shale—was very low. Fortunately, considerable information on the characteristics on this rock was available because some of the structures of the hydroelectric plant already completed were built on closely related strata of shale. Many foundation test data and prototype settlement observations were on hand. Based on these data, the compressive stresses on the foundation were limited to 50 lb per sq in. In addition, careful consideration had to be given to uniform distribution of all loads in order to prevent unequal settlements of different parts. These requirements necessitated a heavy foundation slab over the full basement area, about 250 ft by 180 ft in size. Along the outside edges of the slab a continuous retaining wall supports the yard fill and building walls. The main yard level is 32 ft above the lower part of the basement. The assumed design flood reaches 19 ft above the basement floor level. Therefore, large earth pressure and uplift forces had to be taken into account. The total concrete yardage of the substructure amounts to 26,000 cu yd.

In the design of the substructure, many special conditions presented themselves. For instance, the south wall, because of space limitations, had to be

designed as a gravity retaining wall. In order to prevent cracking of this wall, which was 17 ft thick, expansion joints were necessary, but no expansion provision was contemplated in the steel framing and the brick walls above. The procedure adopted was to build the retaining wall, with expansion joints at about 33-ft intervals, at a very early date. After they had cooled, a band of heavily reinforced concrete was poured as a continuous base for the brick wall above. In this bank 0.8% of reinforcement steel was used and it is free of cracks.

The foundations for the coal pulverizers consist of 6-ft slabs, 57 ft long. In order to localize vibration, these slabs are separated from the surrounding concrete at the sides and at the bottom by a 2-in. layer of a commercial cork insulating mat.

The part of the condenser pit adjacent to the boilers contains a storage tank for 40,000 gal of distilled water at each unit. To prevent any chemical reaction, the concrete surfaces are lined with carbon brick laid with a special commercially prepared mortar. To prevent any uplift under the bricks, the lining is placed over a layer of porous concrete connected by drain pipes to a special sump. Because of the high temperature to which these pits are exposed, all joints around these blocks have cork fillers to allow for expansion.

To prevent any ground water from entering the basement through these joints, two strips of copper seals were used at all contraction joints, and construction joints were sealed by a single strip.

Each of the 60,000-kw turbo-generator units is supported on a reinforced concrete frame which carries the load to a 7-ft mat for foundation load distribution. Resonant vibration and settlements are of major concern for such a machine foundation. In proportioning the foundation mat, an effort was made to obtain equal settlement under each frame column. To prevent the development of resonant vibrations, the special design recommendations of the turbine manufacturer in regard to loading assumptions, deflection limits, and unit stresses were followed closely. The structure designed on this basis is reinforced heavily. The steel amounts to 285 lb per cu yd of concrete. Because of the stipulation of the manufacturer to disregard shear strength of concrete altogether, columns and beams are crowded with stirrups. A definite sequence for placing the different groups of bars and the forms had to be established to facilitate construction of these members.

The results both as to balance of the rotating equipment and as to absence of resonant vibrations are very gratifying. From sensory inspection, vibrations are scarcely noticeable.

For the condenser water supply, special conditions led to the adoption of a somewhat unusual layout. With a site below the dam, the question arose immediately whether the system should provide circulation by pumping water from tailwater below the dam or by a gravity feed from headwater above the dam. On careful investigation, both systems proved feasible and their costs, with proper allowance for power for pumping and for the loss of generation at the hydro plant, respectively, were almost identical. Because of higher dependability, the gravity feed system was adopted.

The hydraulic conditions are illustrated in Fig. 19. A pressure conduit carries the water from the dam to the condenser. The maximum water requirement of each condenser, at full load with summer temperatures, is 156 cu ft per sec. With the station 800 ft away from the river in an area where founda-

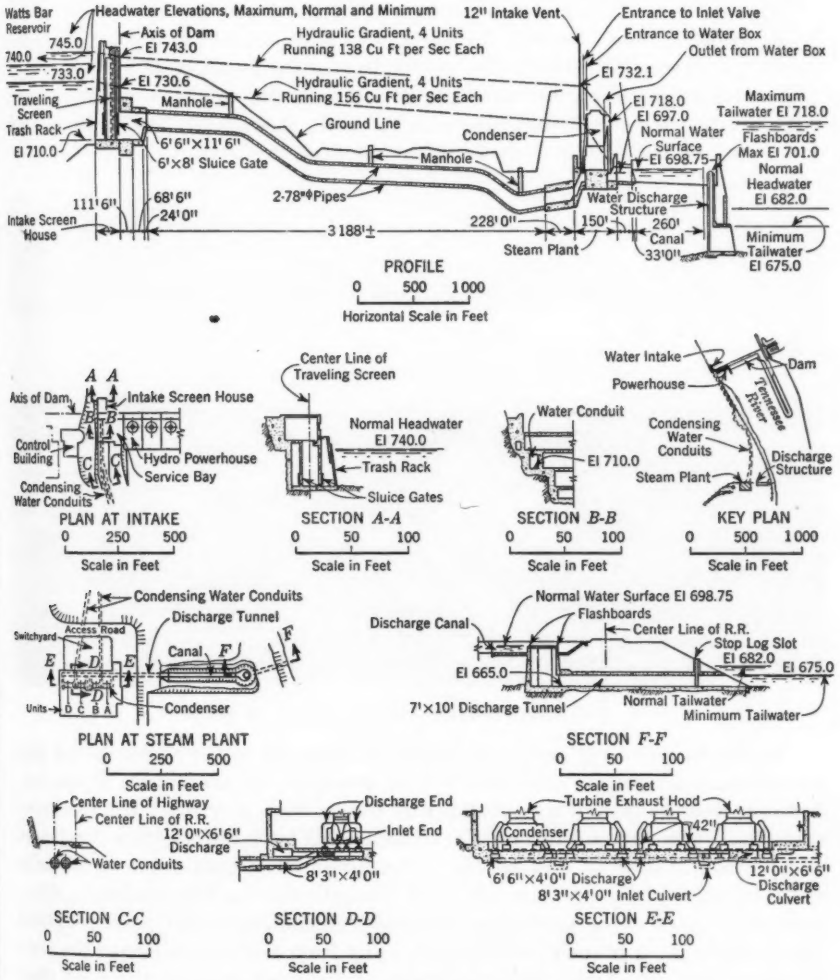



FIG. 19.—CONDENSING WATER SUPPLY SYSTEMS

tion rock is substantially above normal river level, a discharge pool had to be provided at basement level. The available head above this pool, with the forebay at its lowest level, is just sufficient to provide the maximum flow. The hydraulic gradient for this condition is indicated in Fig. 19.

Also shown is a gradient for high tailwater conditions, caused by infrequent floods. The condition shown refers to the assumed design flood, with tailwater

20 ft above the discharge pool. In case of such a flood, the conduit capacity would be reduced to 138 cu ft per sec, but because such floods do not occur during the summer season, the required condenser vacuum could still be maintained.

The intake screen house is located upstream from the shore end of the hydroelectric plant; six screen chambers, with low intakes, connect to a common conduit which leads through the service bay substructure of the hydro plant. Once past the powerhouse area, the conduit divides into two sections, each consisting of a concrete pressure pipe 6.5 ft in diameter. These pipes were fabricated by the pipe manufacturers at the project site.

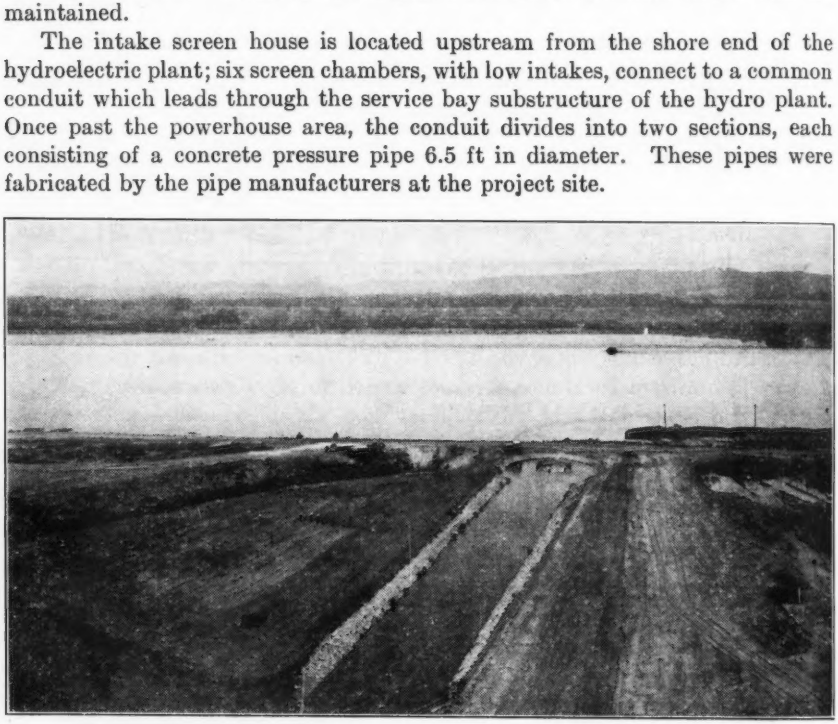


FIG. 20.—DISCHARGE CANAL AND MORNING-GLORY SPILLWAY

Where the conduits enter the heavy fill adjacent to the building, the pipe construction terminates and reinforced concrete boxes are used. These conduits continue as part of the substructure blocks until they reach the condenser entrance. The two entrances are each controlled by a 42-in. gate valve placed in a horizontal position and set for a slow 6-min closure, in order to minimize water hammer in the pressure line. At the outlet end of the condenser, 42-in. butterfly valves allow throttling for pressure regulation and closing against high tailwater. A branch conduit leads away from each condenser to the main discharge conduit which connects directly to the open canal in the flood plain between the station and the river.

To dissipate the energy of the drop between discharge pool and normal river level, a morning-glory spillway discharges into a box culvert built on rock under the access railroad to the hydro station. Fig. 20 is a view from the station roof overlooking the discharge canal from the outlet structure to the morning-glory spillway at the river end.

The service areas around the power station (Fig. 21) could be provided by a balanced earth grading job which involved moving approximately 400,000

cu yd of earth. Facilities for coal handling and for coal storage made the largest demand for yard grading. Coal can be delivered by rail, by truck, or by river barge. For rail delivery a large track loop is formed around the coal storage area with the tracks passing over the central unloading hopper. Ahead of the hopper, storage tracks accommodate one hundred loaded cars. At the hopper the main operating track is equipped with a rotary car dumper. Car pullers move the empty cars out of the hopper building to the head of an incline over which the cars move by gravity to storage tracks, having space for fifty cars. Trucks delivering coal enter from the access road and unload into the same main hopper.

For barge delivery a dock about 700 ft long was built along the river front adjacent to the coal handling area, using steel sheet piling salvaged from cofferdams. The dock has a face approximately 33 ft high above the river bottom. A stationary unloading crane unloads the barges, and dumps the coal into a receiving hopper which feeds a conveyer, approximately 1,250 ft long. This conveyer terminates at the hopper building. From the central hopper coal is delivered by conveyers to a crusher and from there to the bins of the power plant. Coal designated for storage is moved by conveyers and power scraper into and out of the 200,000-ton capacity storage pile.

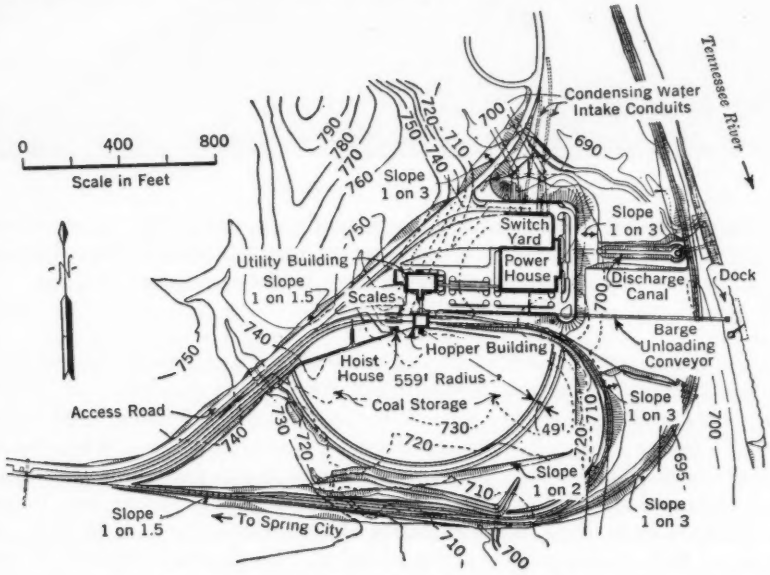



FIG. 21.—GENERAL PLAN, WATTS BAR STEAM PLANT

Careful drainage provisions with ample storm capacity have been made for all the service areas. Care was also taken to obtain stable finished surfaces in all these areas. The slopes exposed to wave action from the river during floods are well protected by riprap. Above the normal flood lines they are protected by grass. In the building area, roads, parking areas, tracks, and grass plots are all well defined by use of concrete curbs.

Like the main building, the yard buildings are all founded on rock. Superstructures were made of light steel frames with walls of asbestos cement boards. The same type of construction was used for the conveyer bridges. Fig. 22 shows the hopper building and its relation to the track loop and the coal storage pile. The rotary car dumper and a truck are shown unloading coal.

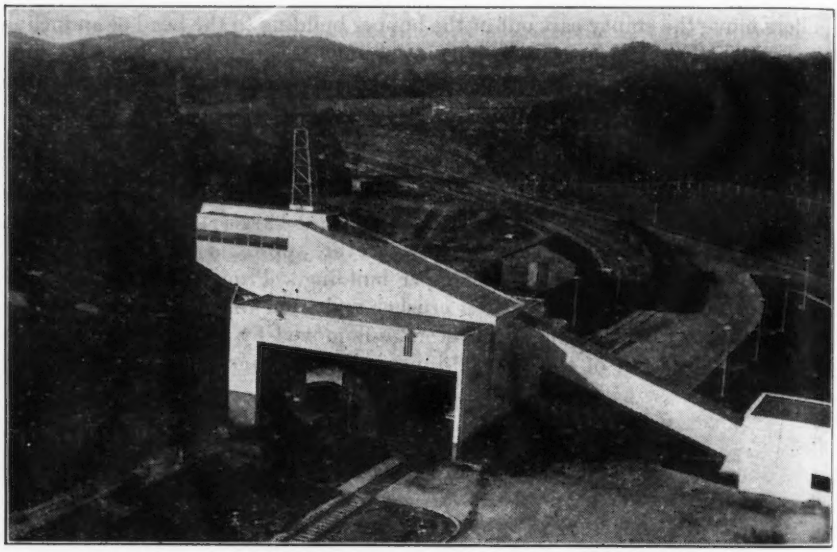


FIG. 22.—COAL STORAGE AREA

An appropriate and pleasing architectural appearance was obtained without any increase in building volume, which was established from functional requirements only and without any changes in the basic steel framing.

## SUMMARY

The Watts Bar Steam Station was initiated as a war plant to furnish power for the armament program during World War II. It was ready at the time when the power was needed and until the summer of 1945 had generated more than 3.5 billion kw-hr of energy available for war production. When the war is over and normal criteria will be applied to measure its usefulness, it will stand up well to such an inquiry. Its capital expenditure is relatively low, the cost per kilowatt amounting to $76 exclusive of transformer yard, or $83 when the electrical tie-in to the switchyard is included. Its thermal efficiency is good; during average operation slightly less than 11,000 British thermal units (Btu) are required to produce one gross kilowatt hour. Coal and ash handling equipment are highly mechanized with resulting low operating cost. Dependability of service was a major endeavor in design and in equipment selection. Experience during three and one-half years of operation has shown very good results.

Now that the war task of the Watts Bar Steam Plant has ended, it can be expected with confidence that, based on the foregoing station characteristics, it will be an economical source of power in peacetime.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# THE EFFECT OF BLANKETS ON SEEPAGE THROUGH PERVIOUS FOUNDATIONS

## Discussion

BY W. J. TURNBULL, AND KENNETH S. LANE

W. J. TURNBULL,[9] M. AM. SOC. C. E.[9a]—Mr. Bennett should be complimented on his modesty in presenting the mathematical analysis of seepage through natural or artificial blankets as not representing an absolutely concise or correct analysis, but rather furnishing a good estimate on which to base computed seepage flows and to determine required length of blankets.

Of all the assumptions which Mr. Bennett makes in his mathematical treatise, there is no question but that the correct determination of actual soil conditions is the critical one and as a result would be the greatest source contributing to errors. Since the accurate determination of the true soil conditions is very necessary for a reasonably good estimate of blanket length, then it becomes all important that every effort be made to obtain reliable and accurate data as to the soil profile and the soils test applicable for each type of soil in the profile. This can be done only by very careful borings, both disturbed and undisturbed. The undisturbed borings are particularly necessary for the proper determination of density and permeability. In the case of cohesionless materials, such as very coarse silts, sands, and sand gravels, it is impossible, of course, to obtain undisturbed samples by boring, and therefore it is necessary to obtain the best disturbed samples possible, particularly with reference to the finer fractions so that none of them are lost, and a true gradation of the material is determined. It is possible to secure relatively undisturbed samples by means of test pits. However, to collect a sufficient number to determine the true average permeability of the pervious stratum is a costly process. In many cases it may be very desirable to conduct large-scale field pumping tests from which the permeability of the pervious stratum may be obtained. The field permeability test has been used very little by engineers in the past. It is believed, however, that this test will become more and

NOTE.—This paper by Preston T. Bennett was published in January, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1945, by Reginald A. Barron; April, 1945, by V. A. Endersby; and June, 1945, by Harry R. Cedergren.

[9] Prin. Engr. Executive Asst., U. S. Waterways Experiment Station, Vicksburg, Miss.

[9a] Received by the Secretary May 16, 1945.

more important and that continued improvement in the technique of the test will result in greater reliability and confidence in the results. There is no question but that such results are much better than laboratory tests on the disturbed material or estimates of permeability made from the grain sizing of the material.

Mr. Bennett presents four conditions to be satisfied before his analysis can be applied with a practical degree of accuracy:

(1) The ratio of the permeability of the pervious stratum to the natural or artificial top stratum is at least ten or greater;
(2) The ratio of the length of the path of travel of the seepage water to the thickness of the pervious stratum is relatively great;
(3) There is only a single pervious stratum which is largely responsible for the underseepage flow; and
(4) The horizontal flow through the embankment structure is relatively small as compared with that through the pervious stratum.

All the foregoing criteria will be met by the greater proportion of embankments, thus making them fall within a category to which Mr. Bennett's analysis can be applied.

If the information on the artificial or natural top stratum, and on the semipervious or pervious layers beneath them, is correct, the results of the analysis demonstrated by Mr. Bennett should be regarded with as much confidence as many other soil tests or design features that comprise the over-all design of an embankment and its drainage facilities. A study of existing embankments and their blankets, assuming that accurate original data were available, would be a valuable contribution to the practicability of Mr. Bennett's analysis. With such a study, one could compare the actual performance record with the estimates derived by the methods of the paper.

The analysis presented in this paper is especially important in that it presents a rational means of estimating blanket length and thickness requirements. Such a procedure may be a means of ending arguments that always arise in the design of an embankment as to the proper length of a natural or artificial blanket of a given thickness and permeability. Many engineers are attaching more and more importance to the design of blankets, because it is believed that properly designed blankets may be used to take the place of the conventional sheet-pile cutoff. Many dams now being built include both the sheet-pile cutoff and a blanket, and once engineers learn to design blankets properly and with confidence, proved by satisfactory performance, the older and more costly, and in many cases ineffective (as far as underseepage is concerned), sheet-pile cutoff may be discarded.

KENNETH S. LANE,[10] ASSOC. M. AM. SOC. C. E.[10a]—The mathematical analysis presented by Mr. Bennett furnishes a relatively rapid method of studying certain seepage problems—particularly those concerning levees and dams across valley flood plains where the soil profile frequently consists of a

[10] Head, Soils Laboratory, U. S. Engr. Office, Providence, R. I.
[10a] Received by the Secretary May 24, 1945.

relatively impervious top stratum or natural blanket overlying pervious sands and gravels. The equations can be solved for uplift pressure or piezometric head acting on the bottom of the top stratum and caused by seepage through the foundation beneath the structure. This uplift pressure on bottom of the top stratum is of particular interest since it is the pressure that causes flotation in the zone of seepage emergence—whenever the upward pressure equals or exceeds the weight of the overlying soil. In the state of flotation either the soil of the top stratum suffers a great loss of shear strength (acting as a heavy liquid with the soil grains held in suspension by the upward seepage forces which equal or exceed the buoyant weight of the grains) or the top stratum may be slightly lifted bodily, followed by some relief of uplift pressure through cracks in the top stratum. In either case, boils frequently ensue. If these boils are not arrested, eventually they may develop into piping that removes the soil from beneath the structure and causes failure. It has been well stated that flotation is the cause and piping the effect; and, if flotation can be prevented, piping cannot occur.[11] Therefore, these mathematical solutions for investigating the uplift pressure causing flotation are felt to be valuable contributions to the methods of seepage analysis.

In developing a theory for relief wells as a treatment for flotation, W. H. Jervis, Assoc. M. Am. Soc. C. E., has obtained a solution for flow through a blanket of finite length which was published in 1939 for limited distribution in a report of the U. S. Engineer Office, Vicksburg (Miss.) District.[12] Mr. Jervis considered the case of flow to a slot of infinite permeability with the boundary conditions shown in Fig. 8. As converted to the notation of the paper, Mr. Jervis' solution is as follows:

For $0 < x < L_1$

$$h_x = h \left[ 1 - \frac{\sinh (a\,x)}{\sinh (a\,L_1) + a\,(L_2 - L_1) \cosh (a\,L_1)} \right] \ldots\ldots\ldots (27)$$

in which

$$a = \sqrt{\frac{k_b}{k_f\,z_b\,z_f}} \ldots\ldots\ldots (28)$$

For $L_1 < x < L_2$, the curve of $h_x$ is a straight line falling to zero at $x = L_2$. The author's solution for these boundary conditions is presented in Eq. 9*a*; but, for reasons given subsequently, it is felt that a denominator term in Eq. 9*a* should read as follows, provided $h_n$ has been correctly interpreted to be defined as shown in Fig. 8:

$$h = h_n \frac{e^{ax} - e^{-ax}}{e^{aL_1} - e^{-aL_1}} \ldots\ldots\ldots (29)$$

Writing Mr. Jervis' solution in terms of head lost ($\Delta h$), as considered by the author:

$$\Delta h = h - h_x = \frac{h \sinh (a\,x)}{\sinh (a\,L_1) + a\,(L_2 - L_1) \cosh (a\,L_1)} \ldots\ldots\ldots (30)$$

[11] "Uplift and Seepage Under Dams on Sand," by L. F. Harza, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1352.

[12] "Underseepage Studies, Black Bayou Levee," by W. H. Jervis, Rept. to U. S. Engr. Office, Vicksburg Dist., Vicksburg, Miss., June, 1939.

at $x = L_1$—

$$\Delta h_{(x=L1)} = h_n = \frac{h \sinh (a\,L_1)}{\sinh (a\,L_1) + a\,(L_2 - L_1) \cosh (a\,L_1)} \quad \ldots\ldots (31)$$

or

$$h = h_n \frac{\sinh (a\,L_1) + a\,(L_2 - L_1) \cosh (a\,L_1)}{\sinh (a\,L_1)} \quad \ldots\ldots (32)$$

Substituting Eq. 32 in Eq. 30:

$$\Delta h = h_n \frac{\sinh (a\,x)}{\sinh (a\,L_1)} \quad \ldots\ldots (33a)$$

or

$$\Delta h = h_n \frac{e^{ax} - e^{-ax}}{e^{aL_1} - e^{-aL_1}} \quad \ldots\ldots (33b)$$

which is the same case as that covered in Eq. 9*a*, using $h = \Delta h$ for lost head.

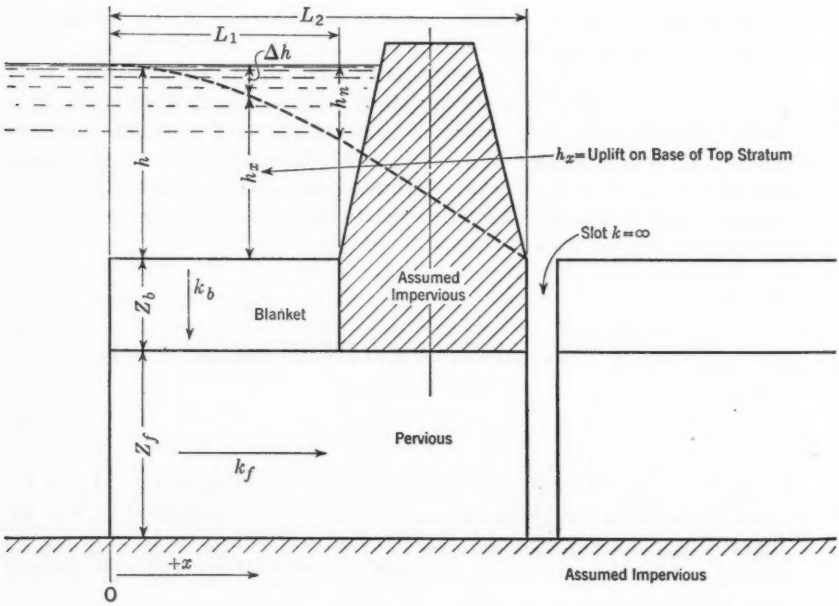



FIG. 8

For $h_n$ defined as in Fig. 8, with this transformation of Mr. Jervis' solution, Eq. 9*a* would read as in Eq. 29 or 33*b*. This is also confirmed by comparison with uplift pressures determined from flow nets and by computations from Eqs. 25.[12a]

It is of interest to compare computations from formulas with uplift curves determined by flow nets. (There are several good references to the utility of

[12a] *Proceedings*, Am. Soc. C. E., March, 1945, p. 409.

Uplift, in Percentage of h
100
80
60
40
20
0
−4b
−3b
−2b
−b
0
+b
+2b
+3b
+4b
+5b
Uplift on Base of Top Stratum
7.3Δh
6.3Δh
5.3Δh
4.3Δh
3.65Δh
3Δh
2Δh
1Δh
Uplift Curve by Formula
Uplift Curve by Flow Net
$N_d = 7.3\Delta h$
$k_1$
$k_2 = 10k_1$
$Z_b = \frac{b}{4}$
$\frac{b}{2}$
$b$
$y$
a
b
c
d
e
f
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
$16 = N_f$
Impervious
1Δh
2Δh
3
0.65Δh
4.3
5.3
6.3

FIG. 9

the flow net.[13,14]) This has been done by using the applicable equations in the paper for four cases of a blanket over a buried pervious deposit. The results are shown in Figs. 9 to 12, inclusive. The two methods of solution give quite close agreement and indicate the validity of the formulas.

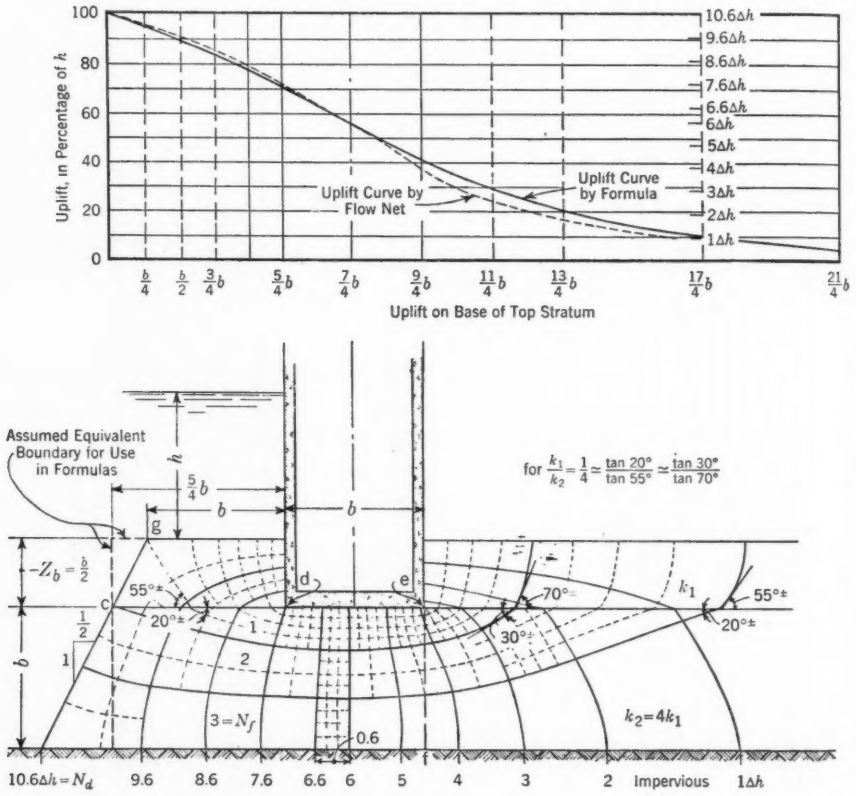


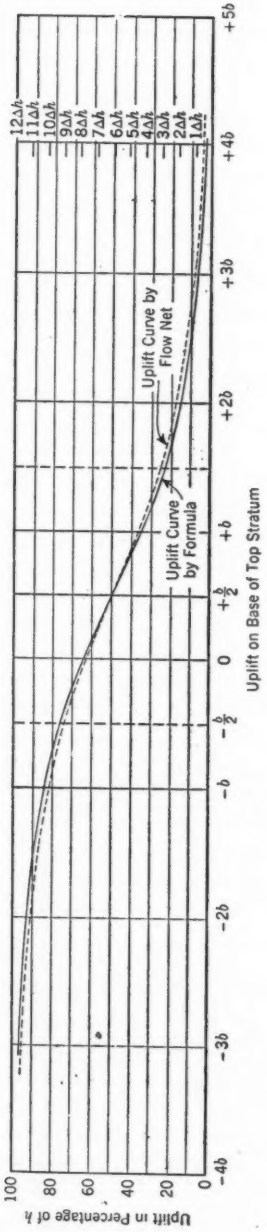

FIG. 10

All the foregoing formulas were derived for flow through a two-layer case (impervious top stratum above buried pervious stratum) and are based on the same differential equation to set up which it was necessary to assume that:

1. The flow is vertical through the top stratum and horizontal through the buried pervious stratum; and
2. No flow occurs in the top stratum under the impervious section of the structure; or, expressed otherwise, the impervious section of the structure extends down to the bottom of the top stratum as shown in Fig. 8.

[13] "Seepage Through Dams," by A. Casagrande, *Journal*, New England Water Works Assn., June, 1937, p. 131.

[14] "Engineering for Dams," by William P. Creager, Joel D. Justin, and Julian Hinds, John Wiley & Sons, Inc., New York, N. Y., Vols. 1 and 3, 1945.


Uplift in Percentage of h
100
80
60
40
20
0
12Δh
11Δh
10Δh
9Δh
8Δh
7Δh
6Δh
5Δh
4Δh
3Δh
2Δh
1Δh
Uplift Curve by Flow Net
Uplift Curve by Formula
-4b
-3b
-2b
-b
-b/2
0
+b/2
+b
+2b
+3b
+4b
+5b
Uplift on Base of Top Stratum

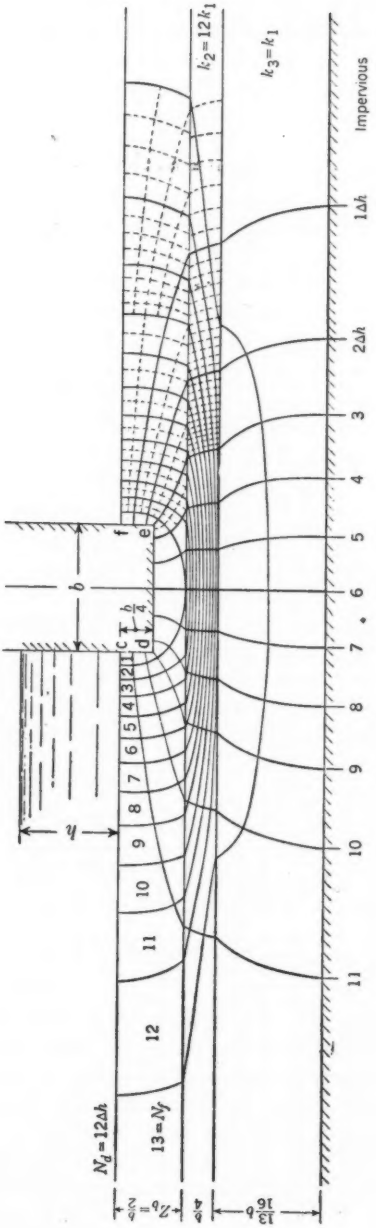

$k_2 = 12k_1$
$k_3 = k_1$
Impervious
$N_d = 12\Delta h$
$13 = N_f$
$Z_b = \frac{13}{16}b$
$\frac{13}{16}b$
$y$
$b$
$\frac{b}{4}$
a
b
c
d
e
f
1
2
3
4
5
6
7
8
9
10
11
12
1Δh
2Δh

FIG. 11

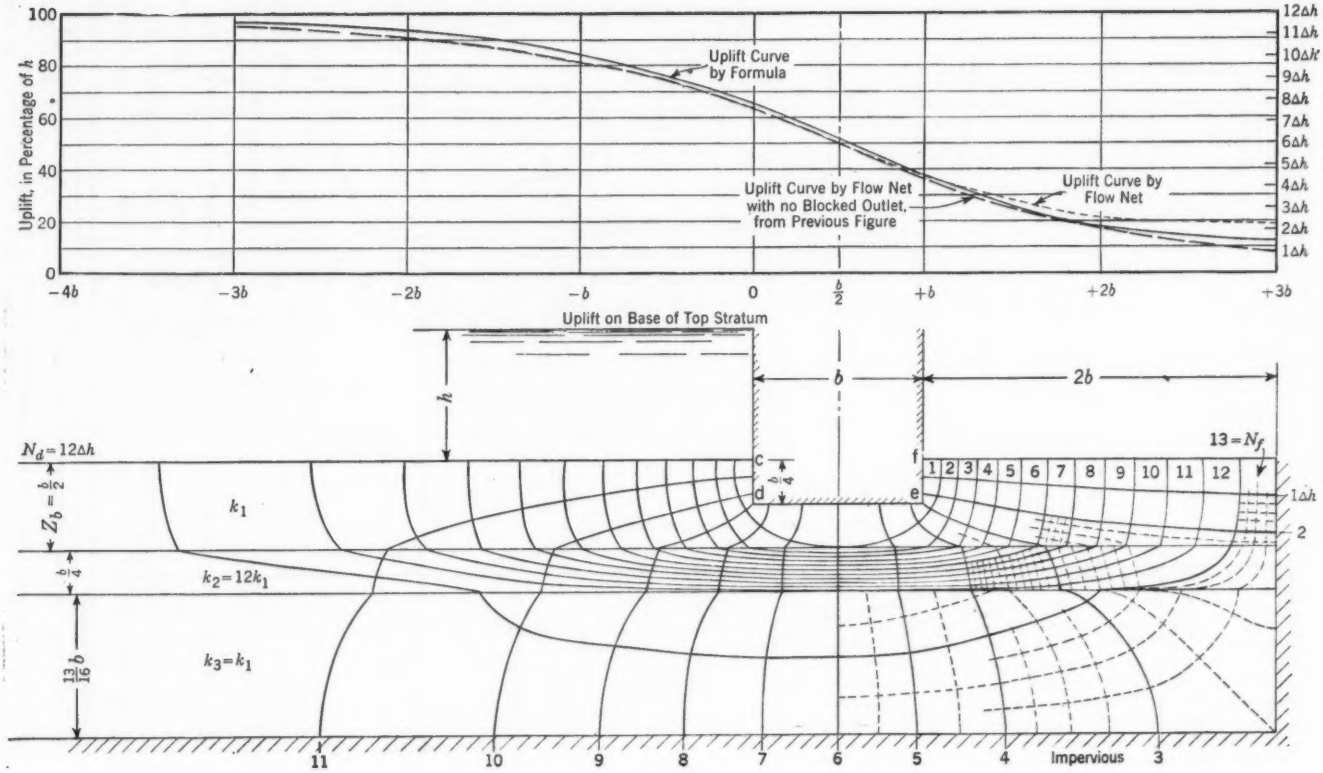

Uplift, in Percentage of h
100
80
60
40
20
0
Uplift Curve by Formula
Uplift Curve by Flow Net with no Blocked Outlet, from Previous Figure
Uplift Curve by Flow Net
12Δh
11Δh
10Δh
9Δh
8Δh
7Δh
6Δh
5Δh
4Δh
3Δh
2Δh
1Δh
−4b
−3b
−2b
−b
0
b/2
+b
+2b
+3b
Uplift on Base of Top Stratum
h
b
2b
N_d = 12Δh
13 = N_f
c
d
e
f
1 2 3 4 5 6 7 8 9 10 11 12
Z_b = b/2
b/4
k_1
k_2 = 12k_1
k_3 = k_1
13/16 b
1Δh
2
11
10
9
8
7
6
5
4
Impervious
3

FIG. 12

Actually, the flow paths are somewhat curved in both the top stratum and the underlying pervious layer, and some flow always occurs in the section of the top stratum beneath the structure; but for practical purposes the foregoing assumptions give results sufficiently accurate for analysis considering the variations in soils and their permeability which are encountered in nature and must be handled by replacing nature's conditions with a simplified or generalized section for the purposes of analysis—a condition which the author has well emphasized. Since they are graphically sketched solutions, flow nets are not precise determinations but rather are good approximations for the chosen boundary conditions. Therefore, although the flow nets of Figs. 9 to 12 are not exact, nevertheless, they give good illustrations of the effect of the foregoing two assumptions which were necessary in deriving the formulas.

Fig. 9 exhibits a flow net solved by K. A. Linell, Jun. Am. Soc. C. E. Here the formula gives a value for uplift slightly too low on the upstream side at point e and slightly too high on the downstream side of the structure at point f—largely because of neglecting the flow through the top stratum in the zone cdef, which in this case is less than one twelfth of the total underseepage. However, the formula is conservative in showing slightly too high a value at point f, as this is the point of most interest in computing the uplift causing flotation in the zone of seepage emergence. Divergence of the formula in the zones from $-b$ to $-3b$ and from $+2b$ to $+4b$ is probably caused by neglecting the curvature of the flow lines in the pervious stratum.

The flow net of Fig. 10 has been drawn for a relatively low ratio of foundation permeability to blanket permeability of 4 to 1, which is likely to be much lower than usual cases in nature. Although the author has warned that the formulas may not be applicable for permeability ratios of less than 10 to 1, the agreement in the uplift curves shown by Fig. 10 is still quite good, and again the formula gives slightly high or conservative results at the point of most interest in the emergence zone, point e (Fig. 10). The formula would agree much more closely with the excess uplift pressure measured in the flow net at point f on the bottom of the pervious stratum as here the flow is horizontal according to the first assumption. The geometry of Fig. 10 exerts a major influence on the flow pattern; and, if the blanket of Fig. 10 had been longer than the relatively short distance of $\frac{5}{4}b$, the agreement between formula and flow net might have been much closer.

The flow net of Fig. 11 covers a three-layer case with a relatively thin pervious layer placed within a deposit which is one twelfth as pervious. Although the formulas are not intended to apply to this case, it appears that they could be used since the check with the flow net is very close. Actually, the influence of the third or lowest stratum is small as it carries only two thirteenths of the total underseepage and the great bulk, or ten thirteenths, of the flow passes through the pervious deposit, leaving only one thirteenth of the flow in the top stratum from points d to e.

In Fig. 12, the upstream half of the flow net is the same as that in Fig. 11, but on the right or downstream half an impervious barrier has been considered at $+3b$ to give a partly blocked outlet. This raises the uplift over the entire

emergence zone as shown by the comparison of the two uplift curves from the flow nets drawn in the upper diagram of Fig. 12. The author's solutions do not cover this case and, therefore, the computations were made from Eqs. 22 and 23, which are considered solutions for a more general case.[14a] At the point of most interest on line ef, the agreement is good; but, in the zones from $+2b$ to $+3b$, the influence of the third stratum becomes important and the use of the formula is not warranted for obtaining the complete uplift curve of this particular three-stratum case.

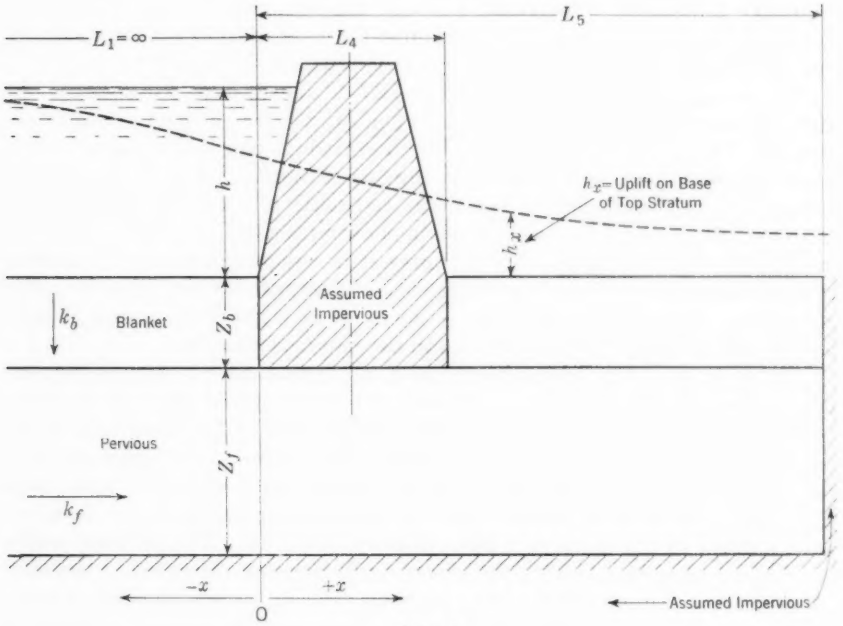



FIG. 13

In using Eqs. 22 and 23 for the solution of Fig. 12, it was necessary to select finite and large values of $L_1$, making the computation rather cumbersome. A further simplification, for the boundary conditions illustrated in Fig. 13, has been derived by R. A. Barron, Jun. Am. Soc. C. E., and is as follows: For $x < 0$,

$$h_x = C\,e^{ax} + h \quad \ldots\ldots (34a)$$

for $0 < x < L_4$,

$$h_x = C + h + C\,a\,x \quad \ldots\ldots (34b)$$

and, for $L_4 < x < L_5$,

$$h_x = D\left[e^{ax} + e^{a(2L_5 - x)}\right] \quad \ldots\ldots (34c)$$

in which the constants,

$$C = \frac{h}{\dfrac{1 + e^{2a(L_5 - L_4)}}{1 - e^{2a(L_5 - L_4)}} - (1 + a\,L_4)} \quad \ldots\ldots (35a)$$

[14a] See *Proceedings*, Am. Soc. C. E., March, 1945, p. 408.

and

$$D = \frac{C}{e^{aL_4}\left[1 - e^{2a(L_8 - L_4)}\right]} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35b)$$

Two advantages of upstream blankets are reduction in uplift pressures in the seepage emergence zone and reduction in quantity of seepage. These apply particularly to dams holding a permanent pool where the blanket is protected by the water above and where the head lost through the blanket exerts a force compressing the blanket and thus increasing its impermeability. The author's use of the expression for $x_r$ as a measure of blanket efficiency offers a quick method of comparing the value of different blanket dimensions.

A triangular blanket is theoretically sound and could be applied where the impervious material is very scarce; but the usual case is likely to be that of a more uniform blanket thickness. In this connection, it is difficult to visualize the author's findings (see paragraph containing Eq. 14) from comparative computations that, for the same area or volume of impervious material, a triangular blanket should be twice as long as one of uniform thickness to provide the same resistance to flow. Any further elaboration the author could inject on this point would be helpful.

In contrast to dams holding a permanent pool, an upstream blanket is not quite as advantageous for flood control dams where water is stored only intermittently with frequent rapid drawdowns of the pool and where the volume of loss from seepage is seldom important. For this use, thicker blankets are apt to be required as the frequent exposures of the blanket during nonstorage periods offer opportunities for erosion and, in the colder climates, for loosening the upper part of the blanket by frost, substantially reducing its impermeability. The latter would apply if the impervious soil of the blanket is subject to frost action as is likely to be the case.

TABLE 2.—PERMEABILITY OF NATURAL BLANKET OVERLYING A GLACIAL DEPOSIT OF PERVIOUS SAND

(All Values Are the Averages of Several Tests)

| Description | Peat | | Topsoil | | Subsoil[a] |
|---|---|---|---|---|---|
| | Undisturbed | Remolded | Undisturbed | Remolded[b] | Remolded |
| Organic content (percentages) | 23.8 | | 6.1 | | 1[c] |
| Specific gravity | 2.25 | | 2.45 | | 2.62 |
| Permeability (cm per sec $\times 10^{-4}$) | 51 | 49 | 87 | 28 | 13 |
| Void ratio | 3.66 | 1.51 | 1.21 | 0.98 | 0.73 |
| Dry density (lb per cu ft) | 30.1 | 56.0 | 69.1 | 77.1 | 94.5 |

[a] Medium sand. [b] Sod removed. [c] Estimated.

The test results in Table 2 were obtained from sampling the surface soils found over a large glacial deposit in northern Connecticut of highly pervious sand (the parent soil), with a permeability of from $100 \times 10^{-4}$ cm per sec to $400 \times 10^{-4}$ cm per sec. The peat represented low-lying swamp deposits, the topsoil represented the top horizon on higher ground, and the subsoil was the

horizon of weathering immediately beneath the topsoil. The high values of permeability may seem surprising at first but they are considered as being caused by the very loose state found in nature and attributed to the annual loosening by frost action. In an impervious blanket for flood control works, it is judged that the upper part of an exposed blanket should be expected to be loosened similarly where frost action is a factor.

A further precaution in using blankets on flood control structures is to avoid placing the blanket in such a position that the pressure of water trapped behind it on drawdown might be sufficient to cause flotation from seepage up through the blanket—often termed a blowout in the blanket. This situation is likely to be encountered more frequently in the case of levees than in the case of dams; and one solution, effective but expensive, is, of course, to weight the blanket down by a cover of pervious material. Another solution, occasionally possible, might be to provide a measure for relieving this back pressure as by avoiding a complete cutoff or by shortening the blanket. Blanket failures from high back pressure of trapped water have been fairly frequent when blankets were extended down over existing slopes, when inclined blankets without sufficient covering weight were placed on dams of large width and poor internal drainage, and when blankets formed canal linings. An outstanding example of piping following a blowout in a blanket is the Memphis (Tenn.) subsidence described by Karl Terzaghi,[15] M. Am. Soc. C. E., although in that case the placing of the blanket down over a river slope was due to nature.

Acknowledgment is made of assistance given by Mr. Barron in checking the computations for the uplift diagrams and by Mrs. R. D. Lacey in preparing the illustrations and making some improvements to the flow nets during tracing.

Corrections for *Transactions:* In January, 1945, *Proceedings*, change "$k_z$" to "$k_b$" in Eq. 4*c* (page 21), Eq. 8 (page 23), and in lines 1 and 2, page 22. Also change "$m$" to "$n$" in Eq. 15 (page 25) and in line 26, page 30.

[15] "Earthslips and Subsidences from Underground Erosion," by Karl Terzaghi, *Engineering News-Record*, July 16, 1931, p. 91.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# A PLAN FOR A MOVABLE DAM

## Discussion

BY L. C. SABIN, AND A. W. SARGENT

L. C. SABIN,[8] HON. M. AM. SOC. C. E.[8a]—A plan for a movable dam based on experience in the construction and operation of canals and locks at Sault Ste. Marie, Mich., is presented in this paper. Although the dimensions assumed and the observations made as to the general problem relate particularly to the conditions there found, the application of the principle is not confined to those conditions. Any criticism that may apply to these specific requirements may not be applicable to less severe situations.

Need of safety appliances to prevent or stop the free flow of water through the canal serving the navigation locks has been in the minds of all engineers who have had the responsibility for the successful and continuous operation of the waterway through the many years of development at Sault Ste. Marie. Conditions have changed as each new unit of facility has been added and the problem thus modified. The variety of devices adopted was not only the result of such changes and of the experience gained in operation of the earlier types, but perhaps these changes have been as much the result of the ideas of the responsible executives in charge at the time.

The situations involved may be considered as presenting two problems, and the first decision to be made is whether both or only one of the requirements is to be met. First, shall dependence be placed on prevention, or second, shall provision be made for stopping the flow once established? If the arrangement is such that the separation of water levels is always made to depend on at least two gates the chances of free communication by reason of a collision with a gate become very small. Such a device has been depended upon in certain situations but it has not for long been the sole dependence at the Sault Canals. Either the inconvenience of operation, or the realization of the possibility and serious result of a failure to properly operate, has resulted in the second requirement; that is, to stop the free flow once established as early as practicable. This is illustrated by a history of the developments at the Sault.

NOTE.—This paper by Isaac DeYoung was published in February, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1945, by C. L. Hall, C. E. Meyerdick, and Ralph R. Leffler; and June, 1945, by Jay L. Southworth.

[8] Vice-Pres., Lake Carriers Assn., Cleveland, Ohio.

[8a] Received by the Secretary June 20, 1945.

The difference in level overcome by the locks at Sault Ste. Marie is normally about 21 ft but varies from 18 ft to 22 ft according to the stages of water in Lake Superior and the lower St. Marys River. It has been estimated that, under conditions existing in some of the canals, velocity may reach from 16 ft per sec to 21 ft per sec in case of communication of levels. The upper level is a canal a mile or more in length, whereas the lower approach is quite short.

The first attempt to provide for the passage of the St. Marys rapids by other than canoes was a lock built (1853–1855) by the State of Michigan having two lifts of about 9 ft each. In connection with this construction a caisson gate was built near the head of the canal. The difficulty of operating this gate was such that about 1859 a set of guard gates, similar to the lock gates then in use, was built just below the caisson gate. For a time, beginning about 1865, these gates were closed whenever a lockage was in progress—that is, when only one pair of lock gates was restraining the upper level. These gates thus became a preventive measure. They probably could not have been closed had a free flow through the canal been established by an accident.

South or landward of the State Locks the single-lift Weitzel Lock was built (1873–1881) with a length of 515 ft, and a width of 80 ft in the chamber and 60 ft at the gates. The Weitzel Lock and the State Locks were served by the same canal, deepened to 16 ft. The State Locks were operated until 1888 when they were destroyed to make way for the next improvement.

The work done in connection with building the Weitzel Lock included a movable dam designed by the late Alfred Noble, M. Am. Soc. C. E. This structure introduced the use of a swing bridge carrying girders and wickets with hand-power machinery for operation. The method of operation was first to close the bridge, the bottom chord resting against the abutments on either side. The iron girders carrying the wooden wickets were then lowered successively against the horizontal sill, the girders turning on the horizontal axis along the lower chord of the bridge. When the girders were in place the wickets, hinged within the girders, were revolved into an upright position closing the canal. This dam was in place until 1896 when the canal was deepened to 25 ft, making the dam no longer operative.

The next change was brought about by the construction (1887–1896) of the Poe Lock which replaced the old State Locks. The Poe Lock is 800 ft long between operating gates. Its original chamber width of 100 ft was reduced to 95 ft in 1942 by placing a 5-ft concrete curtain on the south wall to repair damage caused by blasting for the MacArthur Lock adjacent to it. At first the Poe Lock was served by the old canal deepened to 25 ft and having a width of 108 ft at its narrowest place where it is crossed by the International Railroad Bridge with swing span. To fit this situation a canal gate was first built of the mitering type similar to the lock gates, but constructed of timber and iron.

It was proposed at first to use this as a safety gate, having it closed during any danger period—that is, when the levels were separated by a single lock gate. To serve this purpose only the lower 18 ft was sheathed, the upper part being left open for the passage of water to fill the locks. It soon was realized that delays resulting from such operation would be intolerable in view

of the heavy traffic. Accordingly, it was utilized only as a head gate and fortunately no emergency occurred to call for its closure in a current.

After the completion of the Poe Lock in 1896 the upper canal was widened (1908–1911) giving two channels at the railroad bridge location. These channels, each 108 ft wide, were separated by an island about 80 ft wide carrying the center pier of the bridge which was used also for the center pier of a new movable dam. The closing of the widened and deepened canal, in which the flow in two channels would have to be stopped in case of an emergency, presented a more serious problem and was given much study, including rough plans of many types of structure. The conclusion reached was to build a dam of the swing-bridge type provided with wickets on both leaves.

During the final planning of this dam there occurred, in the canal on the Canadian side, the only accident resulting in the actual communication of water levels. A dam of similar form had been built for this emergency and its use in stopping the flow developed some weaknesses. Advantage was taken of this fact and the plans for the new dam were modified to strengthen certain members.

Each leaf of the bridge carried sixteen steel wicket girders of box cross section about 6 ft 9 in. wide, hung on a horizontal axis at the downstream chord of the bridge and stored under the floor system of the bridge when not in use. When the girders were swung to position, the lower ends resting against the sill, the wickets were released to slide into place. In general form it thus resembled the Alfred Noble dam of 1880. This dam[9] remained in place for about thirty-four years until, in 1944, the deepening of the South Canal in connection with the construction of the MacArthur Lock made it inoperative. Although its use never had been required to stop free flow, it provided insurance and was of some service as a cofferdam in unwatering the canal. Between 1908 and 1919 two new locks were built with a separate canal north of the one to which the preceding description has referred. This north canal has a least width of 280 ft with a depth of 24 ft; the locks and lower approach were built to 24.5 ft below low water datum.

The types of dam considered advantageous were not thought suited to a span of 280 ft and, moreover, it was evidently not desirable to close the entire canal should only one lock be disabled. In the construction of these locks an important innovation was made in providing double service gates at each end. The proposed method of lock operation was such that at no time would the separation of levels depend on one set of gates. Although this is not a certain preventive of level communication it is extremely unlikely that two pairs of gates would be carried away by the collision of a ship. This being still a possibility, however, a movable dam was provided to operate at the upper end of either lock.

This dam consists of an 85-ton bridge capable of serving either the Davis Lock or Sabin Lock and when not in use is stored on the canal wall just above the upper guard gates of the lock. Two large derricks, one on either side, swing this wicket bridge into position in recesses provided in the canal walls. An operating bridge of lighter design is also provided to be placed over the

[9] *Engineering Record*, August 5, 1911, p. 161.

canal about 40 ft upstream from the main bridge to carry a winch truck. Thirty wickets to close the opening, also stored on the canal wall, are placed consecutively by the derrick. A wicket picked up by the derrick is first hung on hinge pins on the wicket bridge; the winch truck moving on a track on the operating bridge then serves to lower the wicket into place against a timber sill at the bottom of the canal. The width of the canal section at the dam is about 90 ft, each wicket closing about 3 ft.

The principal objection to this type is the time consumed in operation. As many as two days have been required for complete closing, although it is thought that this time could be greatly reduced if the operating crew were kept in training. This objection is not as important as might appear, since the principal damage that could occur probably would take place within a few minutes, or at least within an hour after the accident; and no type has been tried which could be operated within that time. However, no traffic in the canal could be resumed until the flow in the damaged lock could be stopped.

The construction of the MacArthur Lock in 1942–1943 presented again the problem of providing means to stop the flow if the wrecking of a lock gate should permit communication of the water levels. This lock replaced the old Weitzel Lock and thus utilizes the south canal serving the Poe Lock.

The plans included the removal of "Bridge Island," thus destroying the movable dam at that location. The resulting width of the canal, with Bridge Island removed, being too great to permit the adoption of any of the types in use, it was decided to place a dam immediately above the new lock and a similar dam at the head of the Poe Lock.

The structure adopted is simple in general plan as it consists of six girders spanning the canal and sliding in vertical grooves in the canal wall. For the MacArthur Lock these girders are about 87 ft long and 6 ft high. The depth of the girders at the center is 6 ft. At the ends they are 3 ft deep and are provided with both reaction and end guide rollers to facilitate sliding to place and removal. The girders are stored on the canal wall and placed by a stationary derrick.

The dam was used as a cofferdam during one winter and considerable difficulty was experienced in removing the girders in the spring because of the formation of ice. Similar trouble might attach to other types if used in winter, but prior steaming should relieve this difficulty. As in the case of other dams its placing in a current has not been tested at the Sault but when not under pressure the time of operation is estimated at two or three hours. A similar structure was later provided at the head of the Poe Lock where the span is about 106 ft, thus requiring much heavier construction.

It may be noted that this stop-log type of dam, properly designed, has very little leakage and thus may serve as a guard gate permitting full access to the regular guard gate of the mitering type.

It is realized that some question may arise as to the justification for the expense incurred in providing the rather elaborate structures described when the probability of an accident that might call for their use has proved so remote; but the volume of the traffic and the effect of any prolonged delay must be considered.

On the completion of the Weitzel Lock in 1881, the first lock built by the United States, the traffic passing the canal the following year was 2 million tons. After the completion of the Poe Lock in 1896 the commerce passing this lock alone in its first full year of operation reached 13 million. In the first full year of operation the Davis Lock in 1915 passed 57 million; the corresponding figure for the Sabin Lock was 51 million in 1920.

A rough value for the cost of the transport concerned may be taken as 10¢ per ton per day. For an annual traffic of 100 million tons, which has been exceeded, this represents a loss of some $40,000 per day and for a delay to one lock a loss proportional to its traffic, except for the amount which may be passed through other locks. Being thus concerned with a commerce volume in excess of the total import and export of all United States ocean and gulf ports, assurance of uninterrupted passage is of such importance as to warrant considerable expenditure.

The author has indicated the relative importance of the desirable features of a movable dam, certainty and reasonable facility of operation being prime requisites, and with this there must be general acquiescence. The dam described has many excellent and novel features and appears to fulfil these prime requirements. The criticism which might be suggested by this discussion relates to the large number of movable parts and the possibility of collecting mud on any submerged structure in a silt-bearing stream. The latter may not be serious in waters of the Great Lakes.

The conservative engineer may wish to see this novel form tested under conditions somewhat less rigorous than those prevailing at Sault Ste. Marie before adopting it for that particular location, where frequent operation is not needed and where a failure to operate when required might prove disastrous. However, for these less rigid conditions it would seem to be well adapted and the author deserves much credit for his contribution in design and description to the general subject of movable dams.

A. W. SARGENT,[10] M. AM. SOC. C. E.[10a]—Some means should be provided at ship canal locks to stop the rush of water through the chamber, in case a lock gate is rammed by a ship or otherwise damaged. The method usually adopted is a dam—an emergency dam—that can be installed in swift water. The type would depend somewhat on the time that should be allowed for placing the dam, and this would vary in different localities. In some canals, the excessive loss of water from the upper pool would seriously affect shipping and the dam should be designed to be placed with the least practical delay. In other localities excessive discharge from the pool would not be of great importance and a less expensive dam, which would require several hours to install, could be used.

An emergency dam must be reliable; it must be ready for use at all times and therefore should be stored where it can be inspected and kept in perfect repair.

---

[10] Stanwood, Wash.

[10a] Received by the Secretary June 21, 1945.

The author proposes an emergency dam located in a sump in the bottom of the lock where inspections and repairs can be made only when the lock is unwatered. Proper maintenance would be very difficult and wire cables could be weakened by corrosion in a short time, especially in locks bordering on sea water. Sunken timbers or other débris could lodge on the dam, preventing its successful operation and, after practice operations of the dam when the lock is not unwatered, there is no visible means of determining whether the dam has settled completely back into place or whether it is partly raised where a vessel could wreck it.

Some of the objections to this type of emergency dam are mentioned in the preceding paragraph. However, at the St. Marys Falls Canal, where the locks are not operated during winter months and there is ample time for repairs, this type of dam may prove satisfactory. It would not be a logical type to use in a lock bordering on sea water, such as Lake Washington Ship Canal, Seattle, Wash., where there is a single large lock for freight traffic which is operated throughout the year except for a very short period for general repairs.

The stop-log type of emergency dam, composed of steel girders placed horizontally across the lock, one on top of another, by a derrick and stored adjacent to the lock when not in use, is a very reliable type and perhaps one of the most costly types for large locks. The type used at St. Marys Falls Canal locks, with all parts stored on the lock walls, and where three to four hours are required to install it, is a satisfactory type at a reasonable cost.

The cost of the structure, or time required for installing, should not be the deciding factor in determining the type of emergency dams for locks. Reliability should be the first consideration.

A skeleton miter lock gate, powered to be safely closed in swift water, after which the complete closure would be made by needles or wickets, might prove a satisfactory type of emergency dam in some locks.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# MOVING LOADS ON RESTRAINED BEAMS AND FRAMES

## Discussion

BY R. W. STEWART, NATHAN D. BRODKIN, AND T. F. HICKERSON

R. W. STEWART,[5] M. AM. SOC. C. E.[5a]—The "transmission coefficient" and "restraint index" used in this paper are special beam and structure constants of a rather complex nature. The "moment of inertia of the loading group," "radius of gyration of a series of loads with fixed spacing," the "span distribution ratio" of the loads; and the "modulus of dissymmetry," "radius of dissymmetry," and "dissymmetry span ratio" constitute quite an array of rather complex functions all relating to the loads.

Using the theorem of three moments as the starting point and deriving the transmission coefficients by the use of Eqs. 1, 2, 3, and 4 involve algebraic work that can be avoided, and which, unless extensively supplemented, is not applicable to structures having members of variable section.

Why not make direct use of the basic constants of flexure[6,7] to obtain the transmission coefficients? This will simplify the analysis, increase the yield of valuable coefficients, and obtain an automatic check of the computations. Fig. 13 will then replace Table 1.

Fig. 13(*a*) shows the torque (998.4) necessary at joint $B_1$ to produce unit rotation at end $E_1$. All moments at the ends of members due to this torque are also shown. This enables not only all transmission coefficients for individual members in a direction away from point $B_1$ to be taken from the figure but it also yields the transmission coefficient from point $B_1$ to any other joint in the frame or from any joint to any other joint which is more remote from point $B_1$. The transmission coefficient across an individual member is obtained by dividing the moment at the end farther from point $B_1$ by the moment at the end nearer point $B_1$. In all cases these agree, to the last decimal place, with

NOTE.—This paper by R. C. Brumfield was published in May, 1945, *Proceedings*.

[5] Bridge Engr., City of Los Angeles, Los Angeles, Calif.

[5a] Received by the Secretary June 27, 1945.

[6] "Relative Flexure Factors for Analyzing Continuous Structures," by Ralph W. Stewart, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 521.

[7] "The Mechanics of Creep for Structural Analysis," by Joseph Marin, *ibid.*, Vol. 108 (1943), p. 477.

the values in Table 1. The transmission coefficient from any joint to any other joint which is more remote from point $B_1$ is obtained by dividing the moment at the more remote joint by the moment at the less remote joint. For example, the transmission coefficient from the left end of member $B_1C_1$ to the top of column $D_1D_0$ is $\frac{9.87}{424.0} = 0.0233$. To obtain the transmission coefficients in the opposite direction a computation is made (Fig. 13(*b*)) beginning with a rotation of unity at end $A_1$ and resulting in a torque of 998.4 at point $D_1$. This agreement with the torque obtained in computing Fig. 13 is an

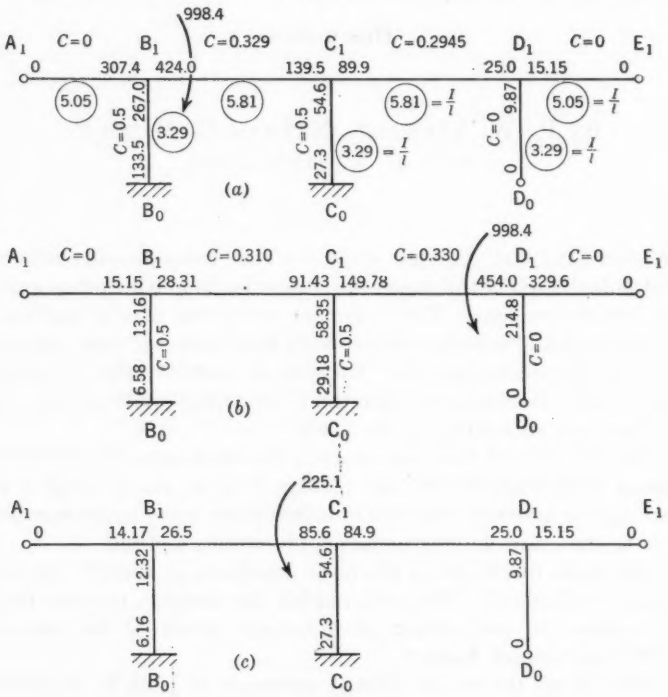



FIG. 13

automatic check by Maxwell's theorem of reciprocal deflections (angular) and indicates that the computations for Figs. 13(*a*) and 13(*b*) are free from error. Fig. 13(*c*) shows the effect of a torque of 225.1 at point $C_1$. The right half of Fig. 13(*c*), including the center pier, is copied from Fig. 13(*a*). The left half is obtained by one setting of a slide rule. The torque of 225.1 is the sum of the resisting moments about joint $C_1$.

The "torque split" and the "moment split" at each joint can also be taken from Fig. 13. These agree with the author's values. A similar quick computation of transmission coefficients and moment and torque splits for a structure involving variable moment of inertia has been published.[8]

[8] "Relative Flexure Factors for Analyzing Continuous Structures," by Ralph W. Stewart, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 527.

The author's computation for placing two wheel-load assemblies on the structure illustrated by Fig. 12, so as to obtain the maximum moment over the center pier, is of mathematical interest. However, it neglects the effect of sidesway. For the problem illustrated in Fig. 12, the effect of sidesway is so small, due to the nearly symmetrical condition of both the structure and the loads about the joint of critical moment, that it can be neglected. If the maximum moment at the top of pier $B_1B_0$ were to be computed, it would be necessary to include the effect of sidesway in order to justify the refinement of finding the exact position of the load for the maximum moment.

The manner in which sidesway can affect the position of loads for maximum moment is illustrated in Fig. 14, which shows two of the complete set of influence

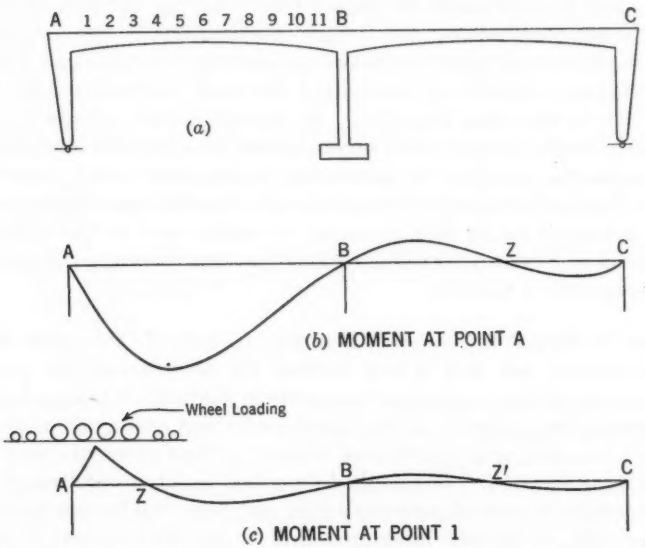



FIG. 14.—INFLUENCE LINES FOR MOMENT, INCLUDING THE EFFECT OF SIDESWAY

lines that were computed for the actual design of a two-span rigid frame bridge of the type shown. In Fig. 14(*b*), the effect of the sidesway is to divide the right span at point Z into two sections in one of which the moment at point A produced by the load is positive and in the other negative. In order to obtain the maximum moment at point A not only span AB but also fractional span ZC would have to have loads properly placed therein. The writer does not find in the paper any means of locating the Z-points (intermediate points of zero influence) in a structure, or of determining whether such points exist. Fig. 14(*c*), which is the influence line for point 1 in Fig. 14(*a*), illustrates another complication which will occur if railway loading is used. The distance AZ is too short to accommodate the wheel base of a locomotive and tender, so that for maximum positive moment at point 1 some of the wheels of the assembly will be at the right of point Z no matter which way the locomotive is faced. A still further contingency which is not infrequent is the reversal of the direction

of the bending moment in a column due to sidesway. How could the author's method of procedure be applied to such a condition?

For a good design of a concrete rigid frame bridge, in which both the maximum positive and maximum negative moments must be computed from point to point in order to properly locate ends of reinforcing bars, the writer sees no escape from the computation of complete influence lines. This can be done in a very moderate length of time by a computer who has a thorough knowledge of the work. There are not very many engineers who would approve constructing the $60,000 structure illustrated in Fig. 14 to a poorly detailed design rather than a well detailed design in order to save from two to three man-days of computing work.

Fortunately the operation of placing trucks having variable spacing on a structure is not required by the design specifications in general use. The truck trains which have been heretofore specified by the American Association of State Highway Officials all have fixed distances between trucks, and the trucks all face in the same direction. At present, either a single truck wheel assembly or a single concentration superimposed on a uniform load is specified. This eliminates the necessity for positioning independent wheel assemblies.

To give the author's method practical value it will be necessary to demonstrate its extension to variable moment of inertia and to the various contingencies due to sidesway introducing Z-points and reversing the direction of bending moment in a member.

NATHAN D. BRODKIN,[9] ASSOC. M. AM. SOC. C. E.[9a]—In this paper Professor Brumfield presents not only a new method for determining the position of moving loads to produce maximum moments in continuous beams, but—what seems of greater importance—he has developed a new and comparatively simple method for analyzing continuous frames so that moments may be distributed in one operation. The method described by Professor Brumfield finds its basis in certain structural concepts which, although well known, have seldom been as logically or lucidly integrated into a practical system of analysis. Utilizing these concepts, the author has developed a method of solution based on determining the exact values of the carry-over factors of the members of a structure.

In a rigid-frame structure, the elastic restraining effects of the various members are interrelated and the constants of the structure, dependent only on this interrelationship, govern the law of distribution of moments throughout the structure. The author terms these constants "frame characteristics" and gives simple equations and a procedure for finding and tabulating them. Distribution of moments from an originating member to the remaining members of the structure then becomes possible in a single arithmetical operation.

The author states in Section 4 that it is "possible to compute, accurately and without the concept of fixed-end moments, the induced restraining moments due to loads placed on the span." This is quite true. However, the writer would like to indicate certain simplifications that can be made in Eqs. 27 for

[9] Civ. Engr., Board of Transportation, New York, N. Y.

[9a] Received by the Secretary August 1, 1945.

use with static loading, such as is usually encountered in the design of building frames, through the introduction into these formulas of the fixed-end moment concept.

From an inspection of Eqs. 26 it is seen that the restraining conditions of the beam ends have no effect on the values of $B$ and $D$, these values remaining constant for any given system of beam loading. Any known condition of restraint may therefore be applied to the ends of the beam, and the values of $B$ and $D$ then may be expressed in terms of such restraint. For convenience, a condition of total restraint will be applied. Under such a condition, $C_L = C_R = 1/2$, and the values of $M_R$ and $M_L$ in Eqs. 27 become the well-known fixed-end moments $M_{fR}$ and $M_{fL}$, respectively. The values of $A_R$ and $A_L$ in Eqs. 28 become 2/3, and Eqs. 27 may then be written in the following form:

$$M_{fR} = -W\,L \times \tfrac{2}{3} \times [B - \epsilon^3 - \tfrac{1}{2}\,(D + \epsilon^3)] = -\frac{W\,L}{3}\,(2\,B - D - 3\,\epsilon^3) \quad (45a)$$

and

$$M_{fL} = -W\,L \times \tfrac{2}{3} \times [D + \epsilon^3 - \tfrac{1}{2}\,(B - \epsilon^3)] = -\frac{W\,L}{3}\,(2\,D - B + 3\,\epsilon^3) \quad (45b)$$

Solving Eqs. 45 simultaneously for $B$ and $D$ yields the following:

$$B = -\frac{2\,M_{fR} + M_{fL}}{W\,L} + \epsilon^3 \quad (46a)$$

and

$$D = -\frac{2\,M_{fL} + M_{fR}}{W\,L} - \epsilon^3 \quad (46b)$$

Eqs. 46 express the values of $B$ and $D$ in terms of the fixed-end moments and the dissymmetry span ratio. When the loading is symmetrically placed on the beam the value of $\epsilon$ becomes zero. Substituting the values of $B$ and $D$ from Eqs. 46 into Eqs. 27 and simplifying, expressions for $M_R$ and $M_L$ in terms of the fixed-end moments $M_f$, and transmission coefficients are obtained as follows:

$$M_R = A_R\,[(2\,M_{fR} + M_{fL}) - C_L\,(2\,M_{fL} + M_{fR})] \quad (47a)$$

and

$$M_L = A_L\,[(2\,M_{fL} + M_{fR}) - C_R\,(2\,M_{fR} + M_{fL})] \quad (47b)$$

When one end of the beam is hinged and one end is restrained, Eqs. 47 reduce to either

$$M_R = 2\,M_{fRH}\,C_R \quad (48a)$$

or

$$M_L = 2\,M_{fLH}\,C_L \quad (48b)$$

in which $M_{fRH}$ and $M_{fLH}$ in Eqs. 48 are the fixed-end moments to be applied to the restrained end, the other end being hinged.

If the loading is symmetrically placed on the span and both ends are restrained, $M_{fR}$ and $M_{fL}$ in Eqs. 47 are equal. Substituting the single value $M_f$ for $M_{fR}$ and $M_{fL}$, Eqs. 47 become

$$M_R = 3\,A_R\,M_f\,(1 - C_L) \quad (49a)$$

and

$$M_L = 3\,A_L\,M_f\,(1 - C_R) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (49b)$$

It should be noted that the values of $\epsilon$ and $\lambda$ do not enter into the solution of Eqs. 47 to 49 for $M_R$ and $M_L$ and need not be computed for static loading. They are, nonetheless, inherent in the values of the fixed-end moments $M_{fR}$ and $M_{fL}$.

It is not intended that Eqs. 47 to 49 be used in place of Eqs. 27 when moving loads are under consideration. In such cases the values of $\epsilon$ and $\lambda$ must be determined for finding maximum moments; but where the values of the fixed-end moments for static loads on building frames have been previously tabulated, Eqs. 47 to 49 afford a quick means for determining $M_R$ and $M_L$. Such tabulations for the usual type of loading normally occurring in buildings may be found in various forms in texts and pamphlets.[10,11]

As an example of the application of Eqs. 47, the bending moments at the supports of beam $B_1C_1$ in the author's illustrative frame problem, with loading as shown in Fig. 12, will be computed. Beam $B_1C_1$ has a span of 30 ft and concentrations of 3 tons and 12 tons located 6.2 ft and 20.2 ft, respectively, from joint $B_1$, the transmission coefficients being $C_L = 0.310$ and $C_R = 0.329$. For this case, $A_L = 0.345$ and $A_R = 0.366$. For a single concentrated load,

$$M_{fL} = -\frac{P\,a\,b^2}{L^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (50a)$$

and

$$M_{fR} = -\frac{P\,a^2\,b}{L^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (50b)$$

in which $a$ is the distance from the left support to the load. With these values inserted in Eqs. 50, the fixed-end moments are as shown in Table 2. Substituting in Eqs. 47:

TABLE 2.—EXAMPLE OF THE APPLICATION OF EQS. 47

| Load $P$ (tons) | POSITION OF LOAD, IN FEET, FROM EITHER END | | FIXED-END MOMENTS, IN FOOT-TONS | |
|---|---|---|---|---|
| | $a$ | $b$ | $M_{fL}$ | $M_{fR}$ |
| 3 | 6.2 | 23.8 | −11.70 | −3.05 |
| 12 | 20.2 | 9.8 | −25.84 | −53.30 |
| Total | .... | .... | −37.54 | −56.35 |

$$M_{1CB} = 0.366\,[(-112.70 - 37.54) - 0.310\,(-75.08 - 56.35)] = -40.1 \text{ ft-tons}$$

and

$$M_{1BC} = 0.345\,[(-75.08 - 56.35) - 0.329\,(-112.70 - 37.54)] = -28.3 \text{ ft-tons}$$

[10] "Continuity in Concrete Building Frames," 3d Ed., Portland Cement Assn., pp. 7–13.

[11] "Analysis of Rigid Frames," by A. Amerikian, U. S. Government Printing Office, Washington, D. C., 1942, pp. 389–392.

These values check with the author's results within the limits of slide-rule accuracy.

It is often necessary and convenient to know the deflection angles at the supports of restrained members. Expressions for the end slopes of a beam may also be derived in terms of the concepts introduced by Professor Brumfield.

From the mechanics of materials the end slopes $\theta_L$ and $\theta_R$ due to a single concentration $P$ are given by the following equations:

$$\theta_L = \frac{Q}{6\,E}(2\,M_L + M_R) + \frac{P\,Q\,L}{6\,E}(2\,k - 3\,k^2 + k^3) \ldots\ldots (51a)$$

and

$$\theta_R = \frac{Q}{6\,E}(2\,M_R + M_L) + \frac{P\,Q\,L}{6\,E}(k - k^3) \ldots\ldots (51b)$$

When the rotation of the tangent to the elastic curve with respect to the normal position of the beam is clockwise at the left support and counterclockwise at the right support, the signs of $\theta_L$ and $\theta_R$ are positive.

Where combination loading is under consideration the quantity $2\,k - 3\,k^2 + k^3$ in Eq. 51*a* may by analogy be replaced by $\bar{k}^3 - 3\,\bar{k}^2 + \bar{k}\,(2 + 3\,\lambda^2) - 3\,\lambda^2 + \epsilon^3$; or $D + \epsilon^3$. Similarly $k - k^3$ in Eq. 51*b* may be replaced by $-\bar{k}^3 + \bar{k}\,(1 - 3\,\lambda^2) - \epsilon^3$; or $B - \epsilon^3$. Eqs. 51 may now be written for combination loading as follows:

$$\theta_L = \frac{Q}{6\,E}(2\,M_L + M_R) + \frac{\Sigma P\,Q\,L}{6\,E}(D + \epsilon^3) \ldots\ldots (52a)$$

and

$$\theta_R = \frac{Q}{6\,E}(2\,M_R + M_L) + \frac{\Sigma P\,Q\,L}{6\,E}(B - \epsilon^3) \ldots\ldots (52b)$$

Substituting the values of $M_R$ and $M_L$ from Eqs. 27 into Eqs. 52, and simplifying, yield the following expressions for the slopes:

$$\theta_L = \frac{\Sigma P\,Q\,L}{6\,E}\{D\,(1 - 2\,A_L + A_R\,C_L) + B\,(2\,A_L\,C_R - A_R) + \epsilon^3\,[A_R\,(1 + C_L) - A_L\,(2 + 2\,C_R) + 1]\} \ldots\ldots (53a)$$

and

$$\theta_R = \frac{\Sigma P\,Q\,L}{6\,E}\{B\,(1 - 2\,A_R + A_L\,C_R) + D\,(2\,A_R\,C_L - A_L) + \epsilon^3\,[-A_L\,(1 + C_R) + A_R\,(2 + 2\,C_L) - 1]\} \ldots\ldots (53b)$$

Eqs. 53 may be written in a simpler form by substituting the values of $B$ and $D$ previously derived in Eqs. 46, yielding the following expressions:

$$\theta_L = \frac{Q}{6\,E}[(2\,A_L - A_R\,C_L - 1)\,(2\,M_{fL} + M_{fR}) + (A_R - 2\,C_R\,A_L)\,(2\,M_{fR} + M_{fL})] \ldots\ldots (54a)$$

and

$$\theta_R = \frac{Q}{6\,E}[(2\,A_R - A_L\,C_R - 1)\,(2\,M_{fR} + M_{fL}) + (A_L - 2\,C_L\,A_R)\,(2\,M_{fL} + M_{fR})] \ldots\ldots (54b)$$

The writer has applied Professor Brumfield's system of analysis in practice to the solution of problems involving restrained beams, frames, and arches of both constant section and varying section. In most cases he has found the author's method more simple and more expeditious than any of the so-called classical methods of solution. The method of analysis presented in the paper can be generalized, formulas can be derived for members of variable section, and it can be shown that the case of constant moment of inertia is simply a special case of the general solution.

T. F. HICKERSON,[12] M. AM. SOC. C. E.[12a]—The "frame characteristics" treated in this paper are derived from the theorem of three moments. Similar frame characteristics introduced by the writer[13] were derived from modified slope-deflection equations in which the "degree of fixity" is introduced. Both methods are theoretically exact, are entirely independent of the loading or deformations (if small), and lead to bending moments at a single operation.

The procedure for moving loads, and fixing their position on the framework to cause maximum bending moments, are treated thoroughly by the author. Necessarily these general bending-moment formulas for any loading combination (Eqs. 25) must be complicated even if they are supplemented with curves. Treating each individual case as a problem in itself (with fewer variables) offers an alternative procedure. One may arrive at a close approximation to the theoretical result after a few trial positions of the loads. This would be a relatively simple procedure. After solving several typical problems one would be in a better position to judge which of the two methods is preferable in practice.

---

[12] Prof. of Applied Math., Univ. of North Carolina, Chapel Hill, N. C.

[12a] Received by the Secretary August 17, 1945.

[13] "Statically Indeterminate Frameworks," by Thomas F. Hickerson, Univ. of North Carolina Press. 2d Ed., 1937, p. 204 (1st Ed., 1934).

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# FLOW IN A CHANNEL OF DEFINITE ROUGHNESS

## Discussion

BY GARBIS H. KEULEGAN

GARBIS H. KEULEGAN,[38] ESQ.[38a]—It is appropriate to discuss briefly the meaning of the coefficients appearing in Eq. 2 which form the basis of the author's working formula, Eq. 7. The characteristic waviness of a smooth wall, which is generally due to imperfections in construction, is expressed by $a_s$. The effect of waviness of the walls may be augmented by standing waves in the flowing water. In any event waviness produces a periodic variation in the thickness of the laminar sublayer. The constant $b$ is a measure of the convection path $l$ at points near the walls. According to experiments the dependence of $l$ upon $z$ (the distance of a point from the wall) is given by $l = z/b$. The Nikuradse value of $b$ for the flow in a circular pipe of constant diameter is 2.5. In divergent channels the value of $b$ varies with the angle of divergence. This fact might suggest that a more rigorous expression for $b$ should contain a term involving the space rate of variation of the shear velocity $V_f$. In all probability the correction obtained in this manner is small and may be safely ignored. The coefficient $\beta$ takes the ratio of depth to width into consideration. The coefficient $\bar{\epsilon}$ is introduced to correct, approximately, the deviations in the velocities imposed by the free surface of the water and by the angles between the walls and the bottom of the channel.

In effect, the author's method of treating the observed data consists of plotting the quantity $V/V_f - b\beta$ against the logarithm of the Reynolds number, the latter being based on the shear velocity. In the event that the accidental deformation of the walls with increase of water depth is negligible, the surface of the water is smooth, the correction coefficient $\bar{\epsilon}$ is sufficiently small, and the locus of observed points is a straight line, then the slope of that line determines the value of $b$. The author establishes in Fig. 9 a straight line which yields a value for $b$ of $3.04 \pm 0.11$. Since this value is at variance with the

NOTE.—This paper by Ralph W. Powell was published in December, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1945, by Joe W. Johnson and E. A. LeRoux.

[38] Physicist, National Bureau of Standards, Washington, D. C.

[38a] Received by the Secretary June 15, 1945.

Nikuradse determinations, the author has concluded that there are either serious systematic errors, or $b$ has values for channels different from those for pipes.

Ignoring the possibility of any systematic errors, the writer desires to examine the effect of a finite value of $\bar{\epsilon}$ on the magnitude of $b$. The consequences of such an effect are not fully discussed by the author. As was mentioned previously, $\bar{\epsilon}$ is supposed to correct for the dynamic differences produced by the free surface of the water and by the angles between the walls and the bottom of the channel. Since the free surface effect is probably small, consider the nonuniform distribution of wall shear produced by the corners in the channel cross section. The shear is minimum at the corners. Denoting the local shear velocity at any point by $v_f$,

$$v_f = V_f (1 + e) \quad \ldots\ldots (21)$$

and $\bar{\epsilon}$ is given by

$$-\bar{\epsilon} = \frac{1}{A} \int e \, dA \quad \ldots\ldots (22)$$

in which $dA$ is an elementary area of the channel cross section and $A$ is the total area.[39]

It is obvious that if $y/B$ is small, the correction due to $\bar{\epsilon}$ may easily be ignored. However, if $y/B$ approaches 0.5, the value of $\bar{\epsilon}$ can be appreciable.

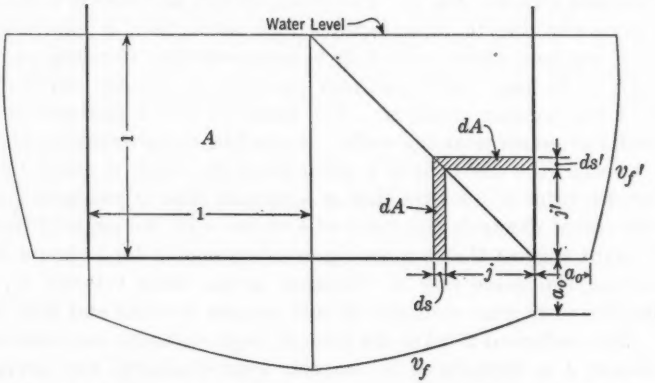



FIG. 15.—DISTRIBUTION OF SHEAR VELOCITY ALONG THE WETTED PERIMETER

By making certain simplifying assumptions the error may be evaluated. Suppose that the width of the channel is 2, the depth of water is 1, and the mean shear velocity is $V_f = 1$ (see Fig. 15). These magnitudes may result by selecting proper units of length and time. The shear velocity distribution along the wetted perimeter may be represented with sufficient accuracy by the expressions

$$v_f = a_0 + a_2 \sin\left(\frac{\pi}{2} j\right) \quad \ldots\ldots (23a)$$

and

$$v'_f = a_0 + a_1 \sin\left(\frac{\pi}{2} j'\right) \quad \ldots\ldots (23b)$$

[39] "Laws of Turbulent Flow in Open Channels," by Garbis H. Keulegan, *Research Paper RP 1151*, National Bureau of Standards (U. S.), Vol. 21, December, 1938, Eq. 46.

of which the first evaluates the shear for the bed, and the second for the side walls. For the meaning of $j$ and $j'$, see Fig. 15. From

$$v_f = 1 + e \quad \ldots\ldots (24)$$

the deviations are

$$e = a_0 + a_2 \sin\left(\frac{\pi}{2} j\right) - 1; \qquad 0 \leqq j \leqq 1 \quad \ldots\ldots (25a)$$

and

$$e' = a_0 + a_1 \sin\left(\frac{\pi}{2} j'\right) - 1; \qquad 0 \leqq j' \leqq 1 \quad \ldots\ldots (25b)$$

Since the average shear velocity along the perimeter is 1, it can be shown readily that

$$\pi a_0 + a_2 + a_1 = \pi \quad \ldots\ldots (26)$$

In the present case, because of symmetry, $A$ may be taken as half of the cross-sectional area and Eq. 22 will be written as

$$-\bar{\epsilon} = \int_0^1 e\, j\, dj + \int_0^1 e'\, j'\, dj' \quad \ldots\ldots (27a)$$

Substituting from Eqs. 25,

$$-\bar{\epsilon} = a_0 - 1 + \frac{4}{\pi^2}(a_2 + a_1) \quad \ldots\ldots (27b)$$

Substituting from Eq. 26,

$$-\bar{\epsilon} = \left(1 - \frac{4}{\pi}\right)(a_0 - 1) \quad \ldots\ldots (28a)$$

or

$$\bar{\epsilon} = 0.274\,(a_0 - 1) \quad \ldots\ldots (28b)$$

Accordingly, the value of $\bar{\epsilon}$ is readily obtained once the shear velocity at a corner is known. Let it be supposed, as a matter of illustration, that, when $y/B = 1/2$, $a_0$ equals 0.5. From Eq. 28*b* it is then seen that $\bar{\epsilon} = -0.13$. If $y/B$ is made smaller, $\bar{\epsilon}$ becomes correspondingly smaller. When the corrections are introduced into Fig. 9, the following changes become apparent. All the points of the diagram are moved downward. The points that were obtained for the greater depths show the greater displacements. Thus, the new line through the corrected positions of the points will have a smaller slope as the result of the unequal displacements of points. Accordingly, the new determination of $b$ may show a smaller disparity from the Nikuradse value. The true modifications can be discussed with certainty only when $a_0$ is determined experimentally.

The thought expressed by the author on the influence of the Froude number is fruitful. Eq. 12 gives his law of resistance for shooting flow in a smooth channel. There is the possibility, however, that the proposed formula is too restricted and therefore in its present form may be misleading. This misgiving has led the writer to the following simple analysis. It may be assumed, as a matter of hypothesis, of course, that the greater resistance encountered in the tests of shooting flow is due to the waviness of the standing waves in the channel.

In the formula for the Chézy coefficient

$$C = J' + m \log_{10} \frac{R}{C} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

$m$ has the same value as in tranquil flow ($m = 40$), but the value of $J'$ is changed through the induced waviness of the surface waters. Values of $J'$ obtained from the data in Table 1($b$) and computed by Eq. 29 are shown in Fig. 16.

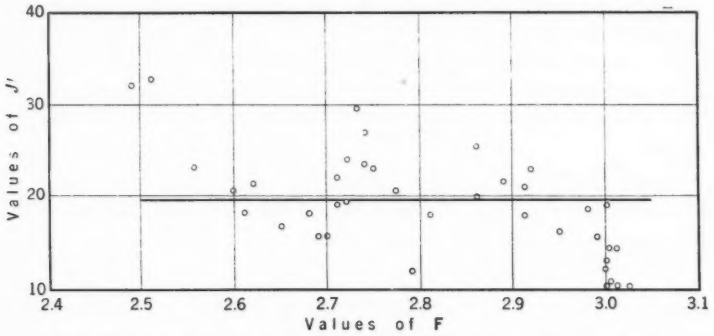



FIG. 16.—RELATIONSHIP OF $J'$ TO FROUDE NUMBER FOR SHOOTING FLOW

As the scatter of the points for the small range of the Froude number considered is pronounced, the line of constant ordinate drawn through the points is judged to be sufficiently representative of the data. This ordinate, which is the average of all the points, has the value of $-19.5$. The resulting resistance law

$$C = -20 + 40 \log_{10} \frac{R}{C} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

is offered as an alternate to the author's law as expressed by Eq. 12. A graph of the quantity $J'$, as obtained from all the tests with the smooth channel, is shown in Fig. 17. The interpretation of the diagram is that, at some critical value of the Froude number, $J'$ changes suddenly, but after the change has occurred its new value probably is independent of the Froude number; or, if $J'$ does depend on the Froude number, the relationship between them is of a nature different from that implied by the author's relation given by Eq. 12. The importance of the problem is great enough to require extensive new experiments on the régime of shooting flow.

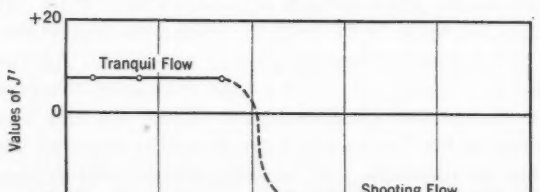



FIG. 17.—RELATIONSHIP OF $J'$ TO FROUDE NUMBER FOR TRANQUIL AND FOR SHOOTING FLOW

The need for various studies in the field of flow in open channels is great. Undoubtedly, men of the profession will feel that the author has made a valuable contribution.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# FUTURE OF LAKE MEAD AND ELEPHANT BUTTE RESERVOIR

## Discussion

BY D. C. BONDURANT, JOHN H. BLISS, LUNA B. LEOPOLD, CARL B. BROWN, AND G. E. P. SMITH

D. C. BONDURANT,[31] ASSOC. M. AM. SOC. C. E.[31a]—The irrigated regions of the United States have been developed to their present economy by reservoirs; and without such reservoirs these regions could not continue to exist in that economy. Since it may be assumed that existing reservoirs have been located at the most satisfactory sites, it follows that they could be replaced only at less satisfactory sites and that ultimately they could not be replaced at all. Unfortunately, reservoir sites are limited and the loss of an existing project may well destroy a community. It is fortunate, indeed, that a few far-sighted individuals have foreseen the seriousness of the problem and have undertaken to bring about its realization.

The writer, however, does not agree with the author that it is not feasible to construct reservoirs upstream from Elephant Butte, N. Mex. It is believed that such reservoirs are not only feasible but are necessary for the continued stability of a large area in the Middle Rio Grande Valley, in New Mexico, supporting about 17% of the population of the state. The problem confronting this region includes not only sedimentation in Elephant Butte but also aggradation of the Rio Grande which is now at or above the level of the valley floor in many places. Consequently, the water table is maintained at such high level that the productivity of the land is seriously impaired due to ineffectiveness of drainage ditches and resulting excessive alkali accumulation. Flood hazards have also increased commensurately. It is possible that aggradation of the Rio Grande is the result of a period of low flow, wherein the flows have been insufficient to carry the sediments brought in by tributary floods and that a later period of high flows may degrade the stream and carry the excess sediments to Elephant Butte Reservoir. It is believed more likely,

NOTE.—This paper by J. C. Stevens was published in May, 1945, *Proceedings*.

[31] U. S. Engr. Office, Albuquerque, N. Mex.

[31a] Received by the Secretary July 16, 1945.

however, that this condition is resultant from irrigation diversions reducing the flow, and the condition will continue as long as irrigation exists.

Reservoir sites are available at one of several points on the main stem in New Mexico, below the Colorado-New Mexico State line, and in the lower reaches of the Rio Chama, Jemez River, and Rio Puerco. These reservoirs could be utilized for flood control, sediment storage, flow regulation, and hydroelectric power. The sediment carried by the streams into these reservoirs would be retained and the clear water released would tend to degrade and stabilize the channel downstream, a premise which is substantiated by observed results below several operating projects. The Rio Grande below Elephant Butte Dam degraded appreciably from Elephant Butte to near El Paso, prior to the channel improvement made by the International Boundary Commission, and it is quite probable that the lack of a heavy sediment contribution from upstream has aided materially in maintaining this improvement. Recent surveys of the South Canadian River below Conchas Dam in New Mexico show that, since 1941, retrogression has been significant for a distance of more than 40 miles downstream. In the Middle Rio Grande Valley, such retrogression would notably improve the status of irrigated lands.

Extensive investigations have been made to determine the feasibility of these projects—both singly and in combination. It has been determined that their construction and operation will in no way abrogate the Rio Grande Compact, but instead will provide a better water supply to the users. The problem of evaporation, as mentioned by the author, is more than compensated by the difference in water surface areas exposed and the difference in evaporation rates at the varied locations. At Elephant Butte evaporation is 5.364 ft per annum computed on a factor of 0.67 to reduce observed pan evaporation to probable reservoir evaporation. At Santa Fe, N. Mex., about 2,500 ft higher in elevation, the reservoir evaporation would be 3.567 ft. The water surface area of Elephant Butte, and consequently the evaporation volume, increases at an appreciably higher rate with storage exceeding 1,000,000 acre-ft than it does with storage less than that amount. During the months of high evaporation, the use of the upstream reservoirs to maintain lower pool elevations in Elephant Butte would result in a net saving of water.

The author's statement that Salt Cedar growth on the Rio Grande is not as extensive as on the Pecos River is correct. However, a dense growth of this plant exists at the head of Elephant Butte Reservoir and the incidence of Salt Cedar growth along the Rio Grande is increasing. This growth should aid in reducing sedimentation in Elephant Butte, although the quantity of water consumed and transpired by the plant is so great that the value of water lost in this manner may offset the efficiency of the plant as a sediment screen.

JOHN H. BLISS,[32] ASSOC. M. AM. SOC. C. E.[32a]—A problem common to practically all storage reservoirs—the reduction and eventual loss of their useful storage capacity due to siltation—is treated in this excellent paper by Mr.

[32] Asst. State Engr., State Engrs. Office, Santa Fe, N. Mex.

[32a] Received by the Secretary July 2, 1945.

Stevens. This is a serious problem, which threatens the very existence of irrigation in the arid west.

In his paper, Mr. Stevens uses the silt data obtained at Elephant Butte Reservoir as the yardstick for determining the unit weight of sediments deposited in reservoirs. Elephant Butte is apparently the only reservoir at which a long-time record of the necessary base data has been kept. Using the 25.75-yr record from January, 1915, through September, 1940, the author derives 65.3 lb per cu ft as the average specific weight of the entire volume of silt deposited in Elephant Butte Reservoir up to the time of the 1940 resurvey. This specific weight is based (1) on the measurements of suspended silt past San Marcial station above the reservoir, adjusted for an estimated bed load and for unmeasured silt flows into the reservoir below the station; and (2) on surveys showing loss of storage space in the reservoir, the volume of silt deposited between San Marcial station and the reservoir, and the loss of dead storage in the reservoir itself. It seems as reasonable a value as may be derived in the light of present knowledge. Any specific weight used depends to a considerable extent upon the bed load that the given stream carries—or is assumed to carry—since no accurate measurements of bed loads are available for the streams in question. The author adjusted the suspended load measurements by increasing them 15% to account for the unsampled bed load. He also deduced, from measurements made on tributaries of the Rio Grande above San Marcial station, that tributary flow contributed an average of about 1,600 tons of silt per square mile of drainage area. Therefore, with 1,750 sq miles of drainage area, the unmeasured tributaries to Elephant Butte Reservoir below San Marcial were assumed to contribute an average of 2.8 million tons per year.

Since no silt measurements were kept for the first eleven years of Elephant Butte operation, the silt inflow past San Marcial for this period was estimated from silt curves. The accuracy of this is questionable. Incidentally, using the author's method of adjustment, the period 1926–1940, when actual silt determinations were taken, yields a value of about 75 lb per cu ft as the unit weight of the deposited sediments. The International Boundary Commission derived a specific weight of 70.2, based on recorded data.[33] Adjustment of this value gives a specific weight for the sediments of well above 80. Part of the discrepancy would seem to lie in the reduced tributary silt inflows which would lessen the bed load carried during this period of below normal runoff. If these latter values are reasonable, it may be that the volume of deposited sediments in reservoirs, particularly over long periods of time, may be somewhat less than that derived by the author. The theoretical life of the reservoirs, in that case, would be extended somewhat, although their eventual demise would be no less certain.

The writer was quite interested in the statement (see first paragraph of Part II) that, although the San Juan, Little Colorado, and Virgin rivers contribute only 20% of the discharge into Lake Mead, they transport 64% of the silt of the Colorado River system above the lake. Here indeed is "an area

[33] *Water Bulletin No. 7*, International Boundary Comm., El Paso, Tex., 1937, p. 63.

where extensive research and consequent improvements would pay handsome dividends."

In his "Conclusions" the author makes a statement regarding the future of Elephant Butte Reservoir which is erroneous and should be corrected. The tri-state Rio Grande Compact does not preclude "any increase in consumptive use of the water in the Upper Rio Grande basin." Rather, it is an agreement between the states that provides the means for making additional and better use of the available water supply by permitting upstream storage and use of water formerly spilled from Elephant Butte Project storage. This is accomplished by converting nonbeneficial uses along the stream to beneficial uses through drainage and reclamation, by permitting development of feasible power projects, etc. Congress has already authorized construction of major reservoirs on Rio Grande and Conejos River in Colorado. New Mexico is currently introducing in Congress a bill for a $40,000,000 project on the Rio Grande and tributaries involving considerable new storage for flood and silt control, power, and re-regulation of the flow through the Middle Valley for irrigation. This project is being integrated by the Bureau of Reclamation and the Corps of Engineers of the U. S. Army and complies, in all respects, with the provisions of the Rio Grande Compact. One of the important features of the proposed project is the detention and control of silt on tributary streams, thereby reducing the average rate of silting of Elephant Butte Reservoir from about 17,000 acre-ft to about 5,000 acre-ft annually. The project will not eliminate the silt problem of the basin, of course, since it merely retains the silt load in reservoirs upstream from Elephant Butte Reservoir. It will alleviate a bad situation for many years, however, during which time it is to be hoped more adequate control measures will become available.

LUNA B. LEOPOLD,[34] JUN. AM. SOC. C. E.[34a]—The Rio Grande and Colorado River watersheds differ somewhat from many other large basins in that normal erosion in the semiarid areas is more widespread and more obvious than in humid areas. Therefore, there is a tendency to minimize the importance of watershed management as a factor that might influence the rates of sediment production.

The author states (see heading, "Prolonging the Life of Lake Mead: Land Management") that in the semiarid areas natural erosional forces are relatively so important that the best land management can have only minor effects in reducing the sediments carried by streams. It should be emphasized, however, that continued mismanagement could increase the sediment available to streams. In a watershed such as the Rio Puerco, an important sediment-producing tributary to the Rio Grande, gullying has proceeded so far that land management alone could have but negligible effect on the lateral widening of the gully walls and the deepening of the existing arroyos in their upper reaches. In the Rio Grande and Colorado watersheds, however, there are myriad small valleys which are not yet gullied and where land management, as a preventive method, can be effective in keeping sediment out of the main channels. Flash

[34] 1st Lt., Army Air Forces Weather Service, Los Angeles, Calif.

[34a] Received by the Secretary July 27, 1945.

flows tend to clog the main channel with debris that might be moved into the reservoirs by larger floods. The larger the area of watershed dissected by arroyos, the larger will be the percentage of storm rainfalls capable of producing flash flows in the tributary channels.

The history of arroyo cutting of the Rio Puerco will serve to illustrate the rapidity with which dissection of such alluvial valleys proceeds and the effect

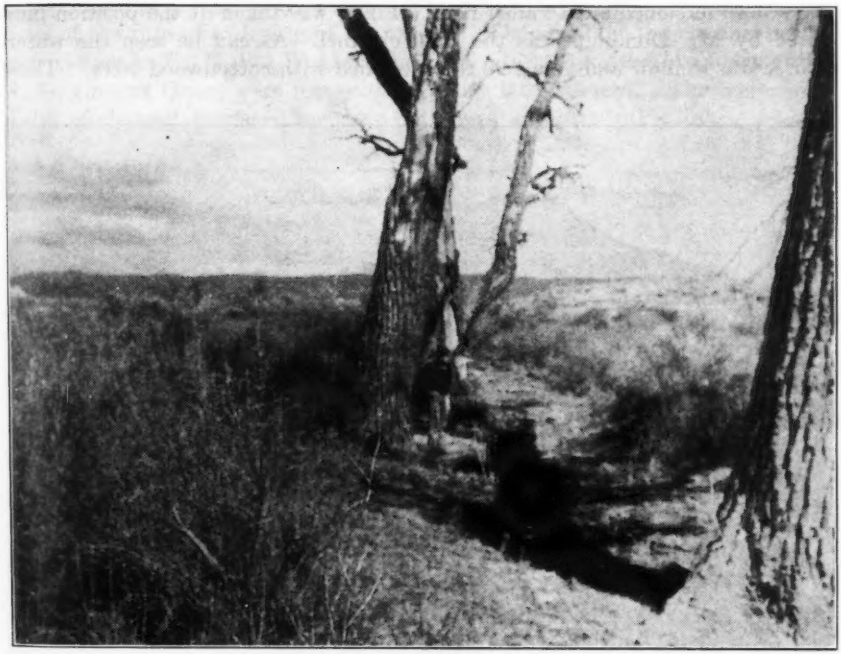


FIG. 8.—OLD STREAM CHANNEL OF THE RIO PUERCO, NEAR SAN LUIS, NEW MEXICO

of recent erosion on the problem of sedimentation of reservoirs. The Rio Puerco comprises 23.6% of the total area above Elephant Butte Dam. The average annual runoff at its mouth[35] amounts to 62,000 acre-ft. Stream-flow records at San Acacia, just below the mouth of the Rio Salado, indicate that the Rio Puerco contributes 6.6% of the runoff and 60% of the suspended load passing that station.[36]

Prior to 1885 the Rio Puerco, an ephemeral stream, was subject to numerous small floods and occasional great ones, the water spreading over a wide and grassy valley floor.[37] The flood plain was discontinuously channeled, a natural feature of such a watercourse. The continuous gully, now approximately

[35] "Rio Puerco Watershed, New Mexico," U. S. D. A. Survey Report, Flood Control, 1941, p. 221 (unpublished).

[36] *Ibid.*, p. 307.

[37] "Erosion and Control of Silt on the Rio Puerco, New Mexico," by Kirk Bryan and G. M. Post, Rept. to the Chf. Engr., Middle Rio Grande Conservancy District, October, 1927.

156 miles long, cut headward from the mouth, the greatest growth taking place in the period from 1885 to 1895.

In 1939 the writer talked to one of the original settlers of the village of San Luis, N. Mex., the late Miguel G. Dominguez, who came there in 1867 when his father settled in the Rio Puerco valley. The old man showed the writer the position of the Puerco channel as it was when he was a youth. As the present arroyo developed, many small remnants of the original channels have been left on the old valley floor. Fig. 8 was taken at the position indicated by Mr. Dominguez as the 1870 channel. As can be seen the watercourse was shallow and about 20 ft wide, lined with cottonwood trees. These

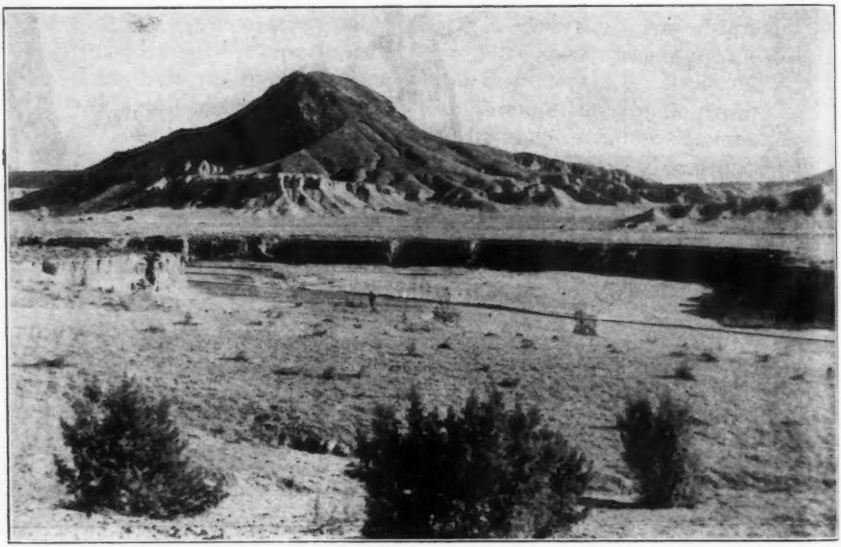


FIG. 9.—ARROYO OF THE RIO PUERCO IN 1940, ABOUT THREE MILES BELOW SAN LUIS, NEW MEXICO

trees died when the water table was lowered by the cutting of the continuous arroyo.

Mr. Dominguez said that in his youth there was a gully "head-cut" 4 ft or 5 ft deep about one mile below the village of San Luis. The gully was probably discontinuous at that time. Such evidence corroborates the picture of the valley constructed by Kirk Bryan and G. M. Post.[37]

In 1904, according to Mr. Dominguez, the arroyo at San Luis was 12 ft deep. It is now 30 ft to 40 ft deep. The magnitude of the present channel can be visualized from Fig. 9, a photograph taken by the writer in 1940.

No discussion of erosion problems of the Rio Grande would be complete without reference to the voluminous work of Kirk Bryan.[38] The classic report by Messrs. Bryan and Post[37] provides detailed data on southwestern erosion

[38] "Historic Evidence on Changes in the Channel of the Rio Puerco," by Kirk Bryan, *Journal of Geology*, Vol. 36, 1928, pp. 265–282.

history. These men made a survey of the Rio Puerco channel in 1927 and another was made in 1939 by the Soil Conservation Service. Using Mr. Bryan's estimate of the volume of the channel in 1885 and the results of the 1939 survey, it is computed that approximately 250,000 acre-ft of material were eroded from the Puerco channel during the 54-yr period.[39]

Data on widths of the Rio Puerco channel recorded by land surveys in 1881, 1897, and 1900 were collected by Messrs. Bryan and Post and, together with later measurements by the Soil Conservation Service,[39] provide indications of the rate of widening of the arroyo. The width of the channel on six section lines in T 6 N, R 1 W where the Puerco forms the west boundary of the N. D. Chavez Grant were measured by early land surveys. The increase of width of channel, averaged for these six places, is shown in Fig. 10.

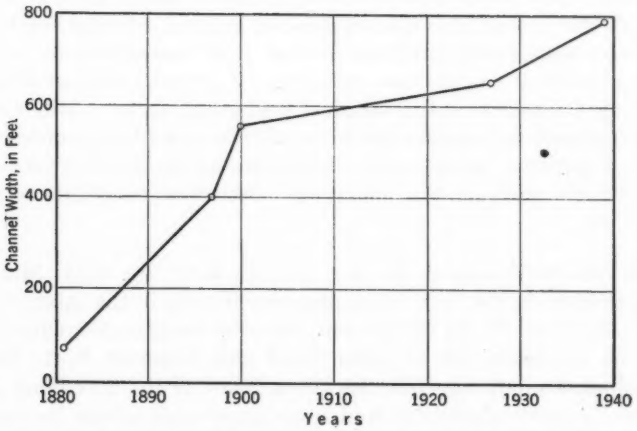



FIG. 10.—INCREASE IN WIDTH OF RIO PUERCO CHANNEL, NEW MEXICO, IN T 6 N, R 1 W

This résumé of the history of arroyo cutting in the Rio Puerco characterizes the development of recent erosion in most of the watersheds of the southwest. This discussion cannot possibly do justice to the large volume of data published on the subject; nor is this the place to discuss the relative importance of overgrazing and climatic change as causes of the present epicycle of erosion. The fact remains that overuse of land has contributed to the timing and to the extent of the present erosion problem. Because there are many subtributaries in various parts of the Rio Grande and Colorado watersheds which are not yet gullied, continued mismanagement of land can still be a factor in producing large amounts of sediment which might further aggrade the main channels or be deposited in reservoirs.

The author states that the prime objective of range and forest management is to produce as much forage and merchantable timber as possible on a sustained yield basis ("Prolonging the Life of Lake Mead: Land Management"). The qualification might well be added that this assumes that the soil and

[39] "Rio Puerco Watershed, New Mexico," U. S. D. A. Survey Report, Flood Control, 1941, p. 218 (unpublished).

vegetation will be ecologically stable under continued use. Such is the case in most areas where deterioration has not progressed too far.

Planting of forests solely to prevent erosion is seldom a sound financial investment, as Mr. Stevens states. This is certainly true in so far as private enterprise is concerned. From the standpoint of the nation as a whole, however, the time has already come when the value of cover on certain watersheds as relatively permanent protection for areas downstream far exceeds the actual cash value of the land itself. It is to be hoped that there will be an increasing realization of the value of watershed cover as protection for water supplies and as a factor promoting soil stability. These are separate values and no less important than the value of the merchantable produce.

The sedimentation problem brings the engineer face to face with many nonengineering aspects of land use. In many respects watershed control, including all the biological considerations of grazing, forestry, and farming, constitutes an engineering problem in that it is susceptible to engineering techniques of over-all analysis and planning. A growing number of engineers are extending themselves into biological fields, but there is still a need for practicing engineers to broaden the scope of their activities in order to apply engineering experience to the vast job of conserving the deteriorating land resource. Papers such as the one under discussion constitute a stimulus toward this end.

CARL B. BROWN,[40] ASSOC. M. AM. SOC. C. E.[40a]—In 1934, Mr. Stevens "focused attention on the part sedimentation will play in the future history of Western civilization." [41] In the present paper he analyzes the effects of sedimentation on the useful life of Lake Mead and Elephant Butte Reservoir. His conclusions strongly re-emphasize the urgency of the task lying ahead of engineers and conservationists to find some practicable means for conserving the limited, irreplaceable, and indispensable reservoir sites on major southwestern streams.

*Lake Mead.*—Previous estimates of the annual rate of silting in a reservoir on the Colorado River at Boulder Canyon have ranged[42,43] from 80,000 acre-ft to 150,000 acre-ft. A rate of 137,000 acre-ft originally estimated by Harry F. Blaney and the late Samuel Fortier, Members, Am. Soc. C. E., in 1928 [44] was quoted by the U. S. Bureau of Reclamation in 1936 with the qualification that "this amount will decrease with upstream development. It is estimated that the total silt deposits in the reservoir will not exceed 3,000,000 acre-feet at the end of 50 years." [45] However, the Bureau's Final Reports on the Boulder Canyon Project contain this statement: "Estimates indicate that before Boulder

[40] Head, Sedimentation Section, SCS, Washington, D. C.

[40a] Received by the Secretary August 1, 1945.

[41] "The Silt Problem," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 208.

[42] "Colorado River and Its Utilization," by E. C. LaRue, *Water Supply Paper No. 395*, U. S. Geological Survey, Government Printing Office, Washington, D. C., 1916, p. 222.

[43] "Handbook of Applied Hydraulics," by C. V. Davis, McGraw-Hill Book Co., Inc., New York, N. Y., 1942, p. 131.

[44] "Silt in the Colorado River and Its Relation to Irrigation," by Samuel Fortier and Harry F. Blaney, *Technical Bulletin No. 67*, U.S.D.A., Washington, D. C., 1928, p. 4.

[45] "Boulder Canyon Project—Questions and Answers," U. S. Bureau of Reclamation, Washington, D. C., May 1, 1936.

Dam was built the average rate of silt transportation through Black Canyon was about 100 thousand acre-feet per year, approximately 5 tons per second." [46]

On the basis of new data, Mr. Stevens has concluded that the average rate of silting in Lake Mead without upstream developments will be 198,000 acre-ft annually. There is considerable evidence to indicate that even this estimate is too low. The author's analysis is based primarily on the suspended load records obtained by the U. S. Geological Survey, at Grand Canyon, during the 16-yr period from October 1, 1925, to September 30, 1941 (see Table 4). His analysis contains no allowance, however, for the fact that this was a period of predominantly subnormal stream flow throughout the Southwest, although he mentions this fact several times in the text. Fig. 4 shows the rapidly declining level of Great Salt Lake and the declining 5-yr moving average of stream flow of the Rio Grande at Otowi, N. Mex., from 1925 through 1935. A cumulative curve of departure from average annual runoff of the Colorado River at Lees Ferry, presented by the writer,[47] shows that the declining trend at this station continued through 1940.

If the quantity of suspended sediment carried by a stream is related to the annual discharge, the long-term mean sediment load would not be fairly represented by a period of exceptionally subnormal flow. The annual suspended load of the Colorado River does have a fairly well-defined exponential relationship to the annual runoff as shown by Fig. 11. As calculated by the method of least squares the curve on this graph was plotted from the equation:

$$S = 0.00295\ R^{1.52} \quad \ldots\ldots\ldots (1)$$

in which $S$ is the annual suspended load in tons; and $R$ is the annual runoff in acre-feet. As the exponent is greater than 1, the suspended-sediment load is shown to increase at a greater rate than the runoff. For some other streams, not in the Southwest, similar calculation has shown an exponent less than 1.

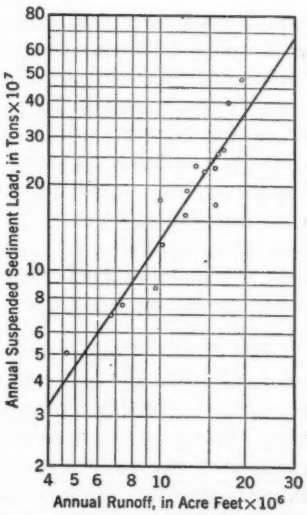



FIG. 11.—RELATION OF ANNUAL RUNOFF AND SUSPENDED SEDIMENT LOAD OF THE COLORADO RIVER NEAR GRAND CANYON, ARIZ., 1925–1941

The spread of points in Fig. 11 is caused mainly by understandable natural conditions. In a year of heavy snow-melt runoff without large summer storms in the silt-producing Colorado plateaus, points may fall well below the line. When the converse is true, points may fall above the line. Over a long period the sum of all measured values should be almost the same as the sum of values derived from the curve, assuming that existing data cover the full range of probable values.

[46] "Boulder Canyon Project—Final Reports," *Bulletin No. 1*, Pt. 4, U. S. Bureau of Reclamation, Denver, Colo., 1941, p. 238.

[47] "Rates of Sediment Production in Southwestern United States," by Carl B. Brown, *SCS-TP-58*, SCS, U.S.D.A., January, 1945.

In order to adjust the mean suspended load of the 16-yr period of predominantly subnormal runoff to a longer, more nearly average, period the following three steps were taken:

(1) An estimate was made of the probable flow at Grand Canyon for each year of the period from 1897 to 1922 by taking a value midway between values estimated by E. B. Debler,[48] M. Am. Soc. C. E., for these years at Boulder Canyon and at Lees Ferry. For a few years certain minor adjustments were made after comparison with estimates made for the same years by E. C. LaRue[49] and R. J. Tipton,[50] M. Am. Soc. C. E. It is considered that Mr. LaRue's estimates of flow at Lees Ferry from 1851 to 1897,[49] based on a correlation with levels in Great Salt Lake, are too uncertain to warrant use in this analysis. If they were used, the indicated mean annual flow would be about 1,000,000 acre-ft larger than that since 1897. Therefore, considering the period from 1897 through 1941 as average seems reasonably conservative by comparison with the 90-yr period since 1850.

(2) Using the estimated runoff at Grand Canyon for each year of the period from 1897 to 1922 in Eq. 1, the value of $S$ for each year was obtained. For the years 1923 to 1925, inclusive, the measured flow was used. The sum of the annual suspended sediment loads thus derived is 8,053,500,000 tons. The measured load during the period from October 1, 1925, to September 30, 1941, was 3,203,900,000 tons, making a grand total of 11,257,400,000 tons.

(3) The measured suspended load during the period from October 1 to December 30, 1925 (approximately 22,000,000 tons, which is duplicated in the preceding addition), was subtracted, and the load for the same months in 1941 (approximately 84,400,000 tons), was added. The record thus derived extends over a period of 45 calendar years, and the mean annual suspended sediment load thus computed is 251,550,000 tons. The computed suspended load for only one year—522,000,000 tons in 1909—was in excess of the 480,000,000 tons actually measured in 1929.

*Bed Load.*—Considering the physical characteristics of the Grand Canyon gaging station, the sediment load that can be measured with existing sampling equipment probably represents at least 95% of the total load passing the station. However, the results of experimentally-controlled tests, and field comparisons of the milk-bottle sampling equipment used at Grand Canyon, with the scientifically designed equipment developed at Iowa Institute of Hydraulic Research,[51] at Iowa City, Iowa, indicate that Mr. Stevens' addition of at least 15% to the reported suspended load is justified to compensate for bias in sampling method plus unmeasured bed load. This percentage addition to the long-

[48] "Stream Flow of Lower Colorado River and Its Tributaries," by E. B. Debler, U. S. Bureau of Reclamation, December, 1934, p. 61 (processed report).

[49] "Water Power and Flood Control of Colorado River Below Green River, Utah," by E. C. LaRue, *Water-Supply Paper No. 556*, U. S. Geological Survey, Government Printing Office, Washington, D. C., 1925, p. 121.

[50] "Report on Water Supply of Colorado River and Allied Matters," by R. J. Tipton, *Exhibit C, Report and Proceedings*, Fact Finding Committee of the Upper Colorado River Basin States, July, 1938 (unpublished).

[51] "Comparative Field Tests of Suspended Sediment Samplers," *Progress Report, Appendix A* of "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," Iowa Inst. of Hydr. Research, Iowa City, Iowa, December, 1944.

term mean value of suspended load, previously derived, would give a mean annual total load at Grand Canyon of 289,282,000 tons.

*Sediment from Intervening Drainage Area.*—The author's estimate of 1,600 tons per sq mile for the annual sediment contribution between the Grand Canyon gaging station and Boulder Dam can be fully substantiated as a long-term average [47] if applied to the area that actually contributes surface runoff. Approximately 9,000 sq miles of the 30,000-sq-mile area between Grand Canyon and Boulder Dam comprises the surface of Lake Mead and areas such as the White River, Upper Meadow Valley, and Upper Las Vegas Valley, which so rarely contribute any water to the Colorado River that they should be excluded as areas of sediment contribution. If the sediment production is estimated as coming from 21,000 sq miles at a rate of 1,600 tons per sq mile, this additional annual contribution of 33,600,000 tons would make the long-term mean annual sediment load entering Lake Mead, in round figures, 323,000,000 tons, rather than the 280,000,000 tons estimated by Mr. Stevens.

*Space Occupied by Deposits.*—The specific weight of deposits in Lake Mead needs to be studied exhaustively. The rate of compaction will affect future operation of the reservoir and should govern the timing of the construction of upstream developments. For example, a mean sediment inflow of 323,000,000 tons (assuming an average specific weight of 65 lb per cu ft) would cause an average annual capacity loss of 228,200 acre-ft. This is equivalent to the loss of the dead storage (3,207,000 acre-ft) plus half of the active storage capacity (12,742,000 acre-ft) to spillway crest (El. 1205.4) in 70 years (from 1935), by which time, according to the author, the usefulness of the reservoir for hold-over storage would be seriously impaired. However, if the final value were only 55 lb per cu ft the period would be 59 years, but at 75 lb per cu ft it would be 81 years.

The effective hold-over storage at any particular time depends on the average specific weight at that time. Practically no usable water can be obtained through compaction of sediment or effluent seepage from a reservoir like Lake Mead in any single seasonal drawdown.

TABLE 10.—RELATION OF SEDIMENT ACCUMULATION TO INFLOW IN ELEPHANT BUTTE RESERVOIR

| Period | Years | Average annual inflow (thousands of acre-ft) | Sediment by volume (%) | Average annual sediment deposit since previous survey (acre-ft) |
|---|---|---|---|---|
| January 1915 to December 1916 | 1.91 | 1,574 | 1.80 | 28,270 |
| December 1916 to August 1920 | 3.67 | 1,414 | 1.66 | 23,437 |
| August 1920 to August 1925 | 5.0 | 1,130 | 1.62 | 18,345 |
| August 1925 to April 1935 | 9.67 | 853 | 1.62 | 13,801 |
| April 1935 to September 1940 | 5.5 | 936 | 0.98 | 9,200 |

*Elephant Butte Reservoir.*—Two factors account for the decline in the rate of sedimentation in Elephant Butte Reservoir. One is the smaller inflow for successive periods between the date of construction and 1935 as shown in Table 10.

Secondly, during the 1937 flood the river broke through its banks above San Marcial and subsequently has deposited huge quantities of sediment over its flood plain. As shown by repeated surveys along eleven ranges crossing the valley between the lower end of the Conservancy District levees and the original head of the reservoir (a distance of about 15 miles), the average annual rate of sedimentation in this area was 50% higher during the period from 1936 to 1941 than during the period from 1914 to 1926. Because of partial desilting of the river flow in this area the rate of deposition in the reservoir was materially lessened.

It is reasonable to expect that the rate of sedimentation in the reservoir will increase when the channel again becomes stabilized in the lower reaches of the Middle Valley and a series of years of heavier flow is experienced. The writer concurs in the opinion that a straight-line extension of the silting rate curve in Fig. 3 "may not be amiss" in the absence of new sediment and water-control structures in the drainage area above.

Under the heading, "Probable Useful Life," the author credits the Soil Conservation Service as reporting "that aggradation of the lower portion of this valley began eight years prior to the construction of Elephant Butte Reservoir." This should be amended to read "at least eight years." [52] Questionable data indicate that the river bed at San Marcial may have been rising since 1880. The Santa Fe Railroad bridge profile at San Marcial, surveyed in 1880, shows a channel 350 ft wide with an average bottom elevation of approximately 4,450 ft. A profile under the bridge in 1904 shows a channel 500 ft wide with an average elevation of 4,452.7 ft.

Aggradation of at least the lower part of the valley since 1906, and perhaps for a much longer period, makes it improbable that the low runoff cycle beginning in the middle 1920's can be the primary cause of this condition. Backwater effects from the reservoir and tamarisk, which the author will find in considerable abundance, have accelerated deposition in the lower reach of the valley, but did not initiate it; nor have they been responsible thus far for increased aggradation more than 25 to 30 miles above the original head of the reservoir.[53]

Although sedimentation in Elephant Butte Reservoir has decreased during successive periods since 1915, coincident with a general decrease in runoff, it does not follow that valley aggradation has resulted from lack of floods capable of flushing out the sediment accumulation of low-water periods.

Records and estimates of the discharge at San Marcial since 1890 show that:

(1) During seven months prior to, and during seven months since, January 1, 1920, the mean monthly discharge has exceeded 10,000 cu ft per sec; and

(2) In ten months prior to, and in ten months since, January 1, 1920, flood flows exceeding 15,000 cu ft per sec have been experienced. The years of largest floods in order of magnitude were 1904, 1929, 1937, and 1941.

In the writer's opinion, aggradation can be accounted for readily by the increase in volume of coarse sediment delivered to the valley by the Rio Puerco

[52] "The Sedimentation in the Middle Rio Grande Valley, New Mexico," by Stafford C. Happ, SCS, 1942, p. 85 (unpublished).

[53] *Civil Engineering*, February, 1945, Fig. 2, p. 84.

and many smaller arroyos that did not exist or were only discontinuous channels prior to 1880. Much of this sediment has come directly from trenching of the alluvial valleys; and, since their develpoment, the arroyos have served as direct flumes for transportation of sediment from gullying slopes into the Rio Grande.

*Prolonging the Life of Southwestern Reservoirs.*—The author's discussion of methods for controlling reservoir sedimentation leaves one with a rather hopeless feeling. The control of density currents has some potentialities, he concedes. Dredging is properly eliminated as economically impracticable. He dismisses land management in the Colorado River basin, and by implication in other arid sections, by expressing doubt (see heading, "Prolonging the Life of Lake Mead: Land Management") that "all the proved practicable methods of land management known today even though lavishly financed" would reduce the sediment load (entering Lake Mead) as much as 10%. By process of elimination, he concludes to spread the sedimentation loss through a series of major reservoirs, which would be constructed according to a schedule based partly on sedimentation rates and partly on the increasing need for stored water. In effect, this plan would throw the burden of sediment control on those very natural resources—the limited and irreplaceable major reservoir sites—that should be protected against, not exhausted by, rapid sedimentation.

One of the most common flaws in reasoning on sediment control in the Southwest results from an assumption that most of the current sediment production is coming (1) from rain-wash erosion of barren slopes that have never supported much, if any, vegetal cover, and (2) from corrasion of rock-bound channels by stream flow. This inference may be readily gained from a cursory look at the spectacular canyons and mesas of the region. All evidence that the writer has been able to collect, however, points toward a contrary conclusion. The present sources of the great bulk of sediment, in order of importance, are the alluvial valley fill, the alluvial fans, and the colluvial and residual soils of the land slopes. In the same order of importance, these were the most favorable sites for, and originally had the densest stands of, vegetation.

When white man first came to know the southwestern country, a cycle of valley and fan alluviation was in progress, except in the canyons of the Colorado River, the lower reaches of some of its tributaries, and in the high mountains. Historical records indicate that channels were lacking or discontinuous in the tributary drainage areas [54] that are now the primary sources of sediment load in the main streams (for example: Rio Puerco and Rio Salado, tributaries of the Rio Grande; Chaco Canyon, tributary of the San Juan; San Simon and San Pedro washes, tributaries of the Gila; Upper Kanab Creek, tributary of the Colorado; Polacca Wash, and other tributaries of the Little Colorado). Erosional debris from the land slopes was deposited mainly on alluvial fans at the edge of valleys or intermontane basins; and intermittent stretches of channel in the valleys terminated in broad areas of water spreading and deposition. At various times in different parts of the Southwest during the nineteenth century, a shift began in the physiographic cycle from predominant alluviation to predominant channeling. Arroyo development has now progressed to the

[54] "Climate and Accelerated Erosion in the Arid and Semi-Arid Southwest, with Special Reference to the Polacca Wash Drainage Basin, Arizona," by C. Warren Thornthwaite, C. F. Stewart Sharpe, and Earl F. Dosch, *Technical Bulletin No. 808*, U.S.D.A., Washington, D. C., 1942, pp. 125–129.

point where most of the western alluvial valleys are cut by deep, steep-sided trenches for most of their length. Branching trenches have extended into many of the tributaries and thousands of gullies run from them on to the land slopes.

A survey of the Rio Puerco made by the Soil Conservation Service in 1939 showed that this arroyo had a volume of 267,000 acre-ft between the mouth and San Luis Dam, a distance of 117 miles. Kirk Bryan estimated in 1928 [55] that the original discontinuous Rio Puerco channels (prior to 1885) contained less than 5% by volume of the then-existing channels. The Soil Conservation Service estimated in 1939 that the current sediment production resulting from enlargement of the main channel of the Rio Puerco was 4,600 acre-ft per yr, or approximately 40% of the estimated average annual load of 11,500 acre-ft passing the mouth of the Puerco. An erosion survey of the entire drainage area gave a basis for estimating that 3,400 acre-ft per yr, or an additional 30% of the total load, was coming from enlargement and headward extension of tributary arroyos; and 3,500 acre-ft per yr, or about 30% of the total, was derived from sheet and gully erosion on the land slopes.

The reduction or elimination of grazing has halted the head cutting of many smaller arroyos and gullies. Proper management of vegetation certainly can retard the acceleration of erosion. Where main drainage channels are already deeply cut, however, vegetation alone cannot control the major part of the sediment output from the watershed. Various types of water-control structures are also essential to sediment control. If the author would broaden his term "land management" to include "upstream engineering" a much different conclusion might be reached. Permanent-type minor structures throughout the watershed can (1) reverse the arroyo cycle now in progress by creating local base levels of erosion, thus forcing water to spread again over valley flats and fan slopes as it did during the cycle of alluviation which existed prior to the present arroyo cycle, and (2) utilize the enormous volume of existing arroyos for sediment storage.

The effectiveness of a barrier system of sediment control has been experimentally tested during a 9-yr period from 1935 to the end of 1943 at the 65,000-acre Navajo Experiment Station of the Soil Conservation Service, situated in the typical rolling plains and broken mesas of the Colorado plateau province, 20 miles north of Gallup, N. Mex.[56] Drainage from this area enters the San Juan River. The experiments were designed to determine (1) the effects of water spreading and sediment deposition on forage and crop yields, and (2) the effectiveness of channel barriers and water spreading in controlling arroyo cutting and reducing the sediment output from the drainage basin of a major southwestern reservoir. All of the experimental area is a direct contributor to sedimentation in Lake Mead.

In 1934 and 1935, fourteen earth-fill dams were constructed to plug the main arroyo channels. Earthen dikes were extended from the dams part way across the adjacent flood plains. The structures forced water out of the channels around the edge of the dikes and caused it to spread over a large area

[55] "Historical Evidence on Changes in the Channel of the Rio Puerco, a Tributary of the Rio Grande in New Mexico," by Kirk Bryan, *Journal of Geology*, Vol. 36, 1928, pp. 265–282.

[56] "Effects of Diverting Sediment-Laden Runoff Water to Crops and Range Lands," by D. S. Hubbell and J. L. Gardner, SCS, April, 1945 (unpublished).

of flood plain. At places on the flood plains where scouring was expected or observed, wire-bound rock sausages were buried or spreader fences were erected.

The percentage of sediment by weight in the runoff from 9,470 acres above three of the fourteen dams, which were chosen for detailed studies, varied from less than 1% to more than 60%, the unweighted average being 13%. In the 9-yr period, approximately 500 acre-ft of sediment was deposited behind the three dams and on the flood plains over which water was spread. The runoff was completely dissipated on the flood plain and no sediment originating above these dams was carried out of the drainage area. The sediment deposits were equivalent to 3.75 acre-ft per sq mile per yr—a high rate of sediment production for even the Colorado plateau area. All of this sediment plus a large additional amount controlled by the other eleven dams would have been added to deposits in Lake Mead.

Complete sediment detention and runoff dissipation can be accomplished by barriers only in small drainage areas. Barriers equipped with overflow spillways can be used effectively, however, in areas up to several thousand square miles. For example, if a series of some 72 such barriers, with spillway sills set at or just above the present valley floor level and with earthen dikes 5 ft to 20 ft high extending to the valley sides, were constructed on a fixed schedule over a 50-yr period on the Rio Puerco, it would be possible:

(*a*) To utilize most of the 267,000 acre-ft channel cavity for sediment storage;
(*b*) To reduce the sediment load of the stream at its mouth by most of the 4,600 acre-ft now coming annually from scour in 117 miles of main channel;
(*c*) To trap approximately 70% of the total sediment output of the drainage basin;
(*d*) To raise the water table in the valley and increase net forage yield; and
(*e*) To retard flood flows.

Such a program would be little, if any, more costly than the construction of a single large reservoir near the mouth of the Puerco to do an equivalent job of sediment control.

The Rio Puerco has an average but highly variable runoff of about 60,000 acre-ft, or less than 7% of the average annual inflow to Elephant Butte Reservoir; yet it contributes more than half of the sediment deposited below its mouth in the valley and reservoir. Loss of 7% of the total inflow to reduce sedimentation in Elephant Butte Reservoir by 50% would be an economic investment. Actually, the control of sedimentation by a watershed conservation program does not involve a sacrifice of water as does the net loss by evaporation from free water surfaces of storage reservoirs. Rather, it would shift the beneficial use of a small fraction of the total water supply of major watersheds (generally in the range of 5% to 15%) to upland slopes and headwater valleys. In compensation it would permit the orderly utilization of major upstream storage sites, such as those enumerated by the author, for beneficial purposes and not for sediment storage.

Mr. Stevens should not have too much misgiving about the efficacy of land management (in its broader sense including upstream engineering) (see "Conclusions") being so closely "bound up with administrative policies and congres-

sional appropriations that its effect * * * lacks the essential element of assurance." It is only 10 years since the effects of land management on agricultural and range land began to be studied on a scale anywhere near commensurate with the scope of the problem. Although the subject is highly complex, enough already has been learned and demonstrated to point the way. Much more research is, indeed, needed to draw the blueprints for full-scale conservation of Western drainage basins. As rapidly as sound plans can be prepared, the afore-mentioned difficulties will doubtless resolve themselves. The will of the local people for conservation of soil and water resources is already expressing itself in the rapid organization under state laws of locally-controlled Soil Conservation Districts which thus far include, for example, 25% of the land in Colorado, 49% in New Mexico, and 68% in Utah.

G. E. P. SMITH,[57] M. AM. SOC. C. E.[57a]—The paper by President Stevens is of absorbing interest. It deserves most earnest consideration in the further development of the comprehensive plan for utilization of the Colorado River, and to a lesser extent of the plans for the Rio Grande.

The rate of silt accumulation in Elephant Butte Reservoir as determined by the author is not greatly different from the rate predicted by the designers at the time of the dedication; the rate is a little less but the difference is accounted for by the difference in the value of $\gamma$. The rate for Lake Mead is greater than was assumed prior to construction of Boulder Dam.

In the realm of history, two or three hundred years is a brief period. Structures and other normal investments can be amortized in a predetermined number of years, but a population settled on land with accessory rural institutions has never yet been amortized successfully. One can picture the farmers on the Colorado River projects clinging to their homes and hoping for exceptionally well-distributed river discharge to enable them to raise a few more crops. Perhaps the present civilization has been too uncharitable in its criticism of the civilization that once flourished in the valleys of the Tigris and Euphrates rivers. In some spots the present civilization is not too secure.

An important query is whether the runoff in quantity and character during those base periods of records available to the author are indeed representative of a far longer period. Since the records of short duration have been extrapolated six to twenty times, it is essential to inquire whether these periods are reliably representative. The writer has studied the rainfall records for Utah, Wyoming, and Colorado, the three states from which runoff at Lees Ferry is derived, from 1880 to 1945. The mean annual rainfall for the three states separately, obtained from the U. S. Weather Bureau, was averaged, weighting the Colorado rainfall double. The record was then smoothed moderately in 3-yr running averages, weighting the middle one double. The resulting record, plotted as percentage of the long-time mean, is shown in Fig. 12. It appears that the rainfall for the years from 1935 to 1942, inclusive, was 105% of the long-time mean. The record in Fig. 12 is not a safe index for the Elephant Butte studies, although a similar one could be made covering rainfall of southeastern Colorado and western New Mexico. It is recognized that runoff is

[57] Prof. of Agri. Eng., Univ. of Arizona, Tucson, Ariz.

[57a] Received by the Secretary August 21, 1945.

not indicated precisely either in quantity or character by rainfall, but rainfall is the best index available and is relied on by hydrologists when missing runoff data must be supplied.

The author refers (see heading, "Probable Useful Life") to "the low runoff cycle the West has been experiencing since 1920." It is difficult to state just what significance is here attached to the much-abused word "cycle." There are wide variations in rainfall and runoff from year to year, but no regular periodic cycles, which repeat themselves over and over indefinitely, can be proved. Even the great irregularities apparent in the rainfall record of a basin, such as the record shown in Fig. 12, do not parallel very well the irregularities of rainfall in adjoining basins. The irregularities are, therefore, largely local; if there were any governing causes, at least they would not be uniform for broad regions like "the West." The vagaries of the air circulation over the earth and of the hairspring conditions that cause precipitation are so great that any truly cyclic tendencies are overshadowed by the sequence of fortuitous events.

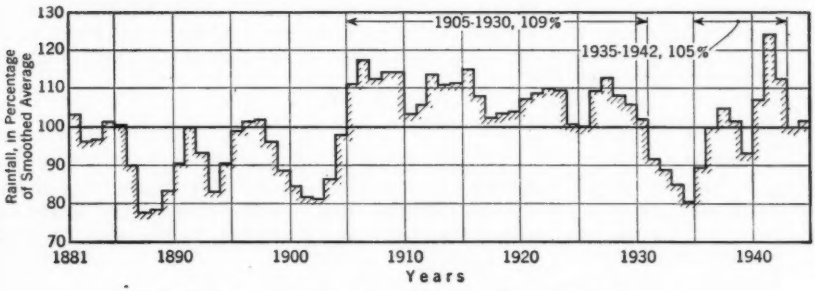



FIG. 12.—RAINFALL IN THE UPPER COLORADO RIVER BASIN STATES

As to the Colorado-Wyoming-Utah rainfall, it is not the drought in the thirties, but it is the generally high level of rainfall from 1905 to 1930 that is outstanding. Other sources of information confirm Fig. 12 and indicate that the long period of low rainfall began in 1870 instead of 1880. The inference is that the author's period of record for the Colorado River, although far too short, is reasonably representative, whereas the period used in the Rio Grande studies covered too many wet years. The importance of a few inches more or less of rainfall in a given year can be appreciated by plotting the quantities of sediment shown in Table 2 against either rainfall or runoff.

The decreasing rate of siltation shown in Table 1 needs further study. A large proportion of the silt comes from the Rio Puerco. Has its bed been aggraded greatly? Also, settling and compaction of the deposited sediments in the reservoir would have the same effect on the rate as decreasing annual increments of silt. The value of $\gamma$ varies within the same reservoir and tends to increase with time.

In the case of Lake Mead, various factors will operate to reduce the siltation rate. Utilization of the water above Lees Ferry, especially construction of projects to utilize the San Juan, will have that effect. Transmountain diver-

sion of clear water, by reducing floods, will tend to aggrade some sections of the river system. The factors are not compensating.

The author proposes, under the heading, "A Schedule of Development," that the Dewey Dam be built first. This suggestion is a mark of real genius. It is desirable that the comprehensive plan for the Colorado River be revised and that the Dewey Dam be made the No. 1 project. That dam and the Bridge Canyon Dam should have been built in 1925 or before, followed by the Davis Dam, and then the Boulder Dam, and dams on the San Juan and Little Colorado rivers.

Two papers by the writer published in 1928 featured the wisdom of building the Dewey Dam first for many reasons. In the first paper[58] it was stated:

> "Canal maintenance in Imperial Valley is made difficult by the deposition of silt. This condition will be greatly ameliorated when the Imperial Canal is extended upstream and connected to the Laguna Dam, as already provided for by agreement. The same methods which protect the Yuma canal system—skimming and sluicing—will be applied to Imperial Canal water at the intake. The silt problem will be greater in the future when storage capacities are threatened than it is now; the silt cannot be stored in reservoirs forever. Revised estimates of the quantity of silt transported annually by the Colorado River are about 50 percent higher than the published estimates."

In the second paper[59] it was stated:

> "Silt storage in the Colorado River reservoirs will suffice for less than two hundred years, and then what shall be done? Silt storage has always been considered a curse. During the next decades, before the water has been put to use and while most of the water will run to the sea, the effort should be to make it carry its load of silt with it, so as to conserve the storage space."

Silt storage as a means of eliminating the silt problem at diversion points, although favored by the farmers of today, won too ready acquiescence of engineers.

The U. S. Bureau of Reclamation is now building projects to irrigate the Yuma Mesa and Imperial East Side, where the soil, or rather the sand, much resembles a sieve. If the naturally silty Colorado River water could be used for 20 or 30 years those lands would be greatly improved; but, ironically, the silt is being stored in Lake Mead where it is not wanted.

The writer often has considered the feasibility of a long tunnel within the rock wall on one side of the canyon upstream from a high dam, at a low level, with inlet tunnels every 200 ft or 300 ft equipped with control gates, but with small permanent openings to bleed off some of the silt. The purpose, of course, is to create a long settling pool and reduce the siltiness of water in the penstocks and turbines. A floating barge with whirling vanes on vertical shafts to be operated seriatim in front of the inlet tunnels might be helpful also. Boulder Dam, more than all others, should have such a tunnel, perhaps one on each side of the canyon extending a mile upstream.

---

[58] *Western Construction News*, February 25, 1928, p. 107.

[59] "Progressive Arizona and the Great Southwest," by G. E. P. Smith, Vol. 9, Nos. 4, 5, and 6 (reprinted in the *Pan-American Geologist*, Vol. LIV, August, 1930)

The author was confronted with inadequate and uncertain data, and has had to make the most of that situation. Records were too short, and silt sampling probably was done during high water or when convenient. He has exercised much license in extensions of data, and in various assumptions, including the various values of $\gamma$, and some factors have been neglected. It is a highly exploratory study; but the computations lead to two definite answers—158 years for Elephant Butte Reservoir and 144 years for Lake Mead.

These answers will be cited and bruited, not only in technical papers, but also in newspapers and in speech until many people accept them as irrefutably exact. In one instance the author refers to his studies as "speculations." It would have been wiser to state the answers as of the order of 158 years and of the order of 144 years, or to have predicted, roughly, about 150 years, and thus leave the impression of uncertainty in the reader's mind. A case in point is that of the preliminary synthetic estimates of the water supply of the Colorado River. It is known now that those estimates were too high; but they were generally accepted to the last significant figure and much damage and injustice have resulted.

Corrections for *Transactions:* Referring to the May, 1945, *Proceedings*, the following corrections will be made in the Stevens paper before the final printing in *Transactions*—on page 605, line 10, change "mitigate" to "militate"; in Table 1, line 3, last column, change "5.40" to "5.30"; on page 606, line 17, make "survey" plural and delete "of 1935"; in Table 2, Col. 2, insert the number of samples for five years (1918, 13; 1919, 82; 1920, 111; 1921, 81; and 1922, 12); on page 607, line 8, after "1913," add "(824 determinations, 1905–1912); in Table 3, heading of Col. 9, change "(acres)" to "(sq miles)", in Col. 1 change "288" to "228," and in Cols. 6 and 9 change "9,520" to "9,460"; on page 610, line 29, change "120" to "130"; in Fig. 5, change "Colorada" to "Colorado" in two places and "Kanah" to "Kanab Cr."; in Table 6, the heading of Col. 3 should be "Period (inclusive)"; in line 1, Table 6, change "63.07" to "63.7"; in line 3, Table 6, change "202.0" to "201.9"; on page 617, line 6, change "1,136" to "1,014"; on page 617, line 8, change "1,485" to "1,363"; on page 621, line 43, omit the parentheses in item 5; on page 625, line 5, change "43 trillion cubical" to "44 billion cubic"; on page 625, line 6, change "1%" to "1¢" and "430" to "440"; on page 625, line 7, change "six" to "four and one-half" and "the entire Boulder Canyon Project" to "the project"; on page 625, line 32, change "2048" to "2248"; and in footnotes 3, 13, 18, and 19 change "E. M. Eakin" to "Henry M. Eakin."

The following changes are to be made on page 618: In Table 8, under the heading "7.66 yr[a]," items 1 to 7 should be, respectively—1,363, 204, 368, 1,935, 8, 1,927, and 252; delete the parentheses in items 7, Table 8; and in footnote [b], Table 8, change "1926" to "1925"; in line 16 change "as demonstrated by" to "in the third column of" and "A summary" to "A similar summary"; in line 17, change "similar to" to "as shown in the fourth column of Table 8" and "1926" to "1925"; and, in line 19, delete "(see Table 4 and item (*a*))."

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# ECONOMICS OF PHOTOGRAMMETRY

## Discussion

BY GEORGE D. WHITMORE, BENJAMIN A. WASIL, CHARLES H. DAVEY, AND ROBERT H. RANDALL

GEORGE D. WHITMORE,[6] M. AM. SOC. C. E.[6a]—This discussion of Mr. Eliel's excellent and comprehensive paper is limited to that part which deals with comparisons and relations between contour mapping by photogrammetric methods and by ground-survey methods.

Mr. Eliel's explanations may leave an impression, with those who are not professional mappers, that a modern contour map may be completely and finally compiled by stereoscopic plotting instruments, without recourse to any follow-up ground surveys. The topographic mapping program of the Tennessee Valley Authority, directed by its Maps and Surveys Division, has included standard contour maps prepared by all three of the principal processes Mr. Eliel describes—that is, (1) multiples, (2) stereoplanigraph, and (3) Brock and Weymouth. This experience makes it seem virtually mandatory that all contour maps compiled by stereophotogrammetric processes be field inspected and field checked. Otherwise, there is no positive assurance that the map as drawn in the photogrammetric laboratory does correctly represent the terrain, and unless certain field checks are made as to contour accuracy, there can never be any real assurance that the stereoscopically compiled contours are accurate within specifications. Thus, it would seem proper to state that, except in emergencies or other rare occasions, the photogrammetric mapping procedures should always provide for the follow-up field inspection and field checking. Accuracy checks are often obtained by random transit and vertical-angle profiles, by numerous side "shots" from a few well-selected plane-table setups, and by other means.

Incidentally, the field engineers who are responsible for field inspecting and field checking operations, commonly called field completion, require as much training and skill as do plane-table topographers. Hence, the skilled

NOTE.—This paper by Leon T. Eliel was published in March, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1945, by Raymond A. Hill, Raymond L. Moore and Ralph S. Pearse.

[6] Prin. Civ. Engr., Maps and Surveys Div., TVA, Chattanooga, Tenn.

[6a] Received by the Secretary July 21, 1945.

topographers, who at one time may have thought that photogrammetry would force them "out of business," instead will probably be more in demand, and for work that should be more interesting.

Another matter not mentioned by Mr. Eliel, but obvious to most experienced mappers, is that probably in most cases a combination, in some degree or other, of photogrammetric contouring and ground-survey contouring will be best. For example, most map sheets will include certain flat or relatively flat areas. It is well known to experienced photogrammetrists that, with present equipment, these flat areas quite often present a difficult problem—that is, it is relatively more difficult to obtain satisfactory contours in flat areas than in rugged terrain by stereophotogrammetric instruments (unless special lower-altitude photographs are provided for those areas). On the other hand, such areas are usually most important economically and, therefore, extra precautions should be taken to make sure that the contours in such areas are relatively more exact. Fortunately, in this case, what is difficult for the stereoscopic plotting instruments is easy for the plane-table topographer. Therefore, many experienced mappers today choose this combination method—that is, the "rolling to steep" terrain to be contoured by the stereoscopic plotting instruments; the flat terrain to be contoured by plane table, usually during the later field inspection and field checking processes.

Another well-known combination of photogrammetric and ground-survey methods that has proved efficient on large projects is that of first preparing a detailed planimetric manuscript base sheet by stereophotogrammetry, on which all contours and elevations are later added in the field by plane-table topographers. After the planimetric base sheet is completed in the laboratory, it is usually reduced photographically to the field scale, the topographer being furnished a blue-line print on drawing paper mounted on aluminum or other scale-holding material. This eliminates the need for long plane-table traverses, and usually results in more accurate planimetry with respect to horizontal position. Of course, it is especially advantageous and economical in flat or gently rolling terrain, and where contour intervals of 10 ft or less are required.

Mr. Eliel did not mention the difficulty of obtaining accurate contours through heavily wooded areas by photogrammetric processes. Densely wooded terrain represents one of the unsolved problems of photogrammetric contour mapping. If the pictures are taken during summer, with the trees tall and closely spaced, and the ground surface irregular beneath the trees, it is extremely difficult for the photogrammetrist to compile standard accuracy contours. He can do better if the pictures are taken during the winter, when the leaves are off the trees, and when snow is not too deep on the ground. Obviously, however, it will be a difficult matter to schedule extensive flying in northern latitudes for satisfactory winter pictures, because of the predominance of cloudy weather during the winter season, because the flying days are short and the shadows long, and because of risk of deep snow. Also, regardless of latitude, an evergreen forest photographs no better in winter than in summer. Unfortunately, ground-survey plane-table methods are also at a disadvantage in mapping contours in wooded areas. It is true that any desired degree of

accuracy can be obtained by ground-survey methods, but the costs accelerate rapidly. The development of a procedure for obtaining accurate contours in densely wooded terrain is the real challenge to photogrammetry today.

BENJAMIN A. WASIL,[7] JUN. AM. SOC. C. E.[7a]—The problems of present-day photogrammetric economics are presented admirably in this paper, which affords an opportunity to look into the future—into postwar mapping—and to appraise the war mapping program as it applies to the future possibilities of making a cheaper map having the same standards of accuracy.

The cost of field control is a major portion of the total cost of a map. Any reduction in field control necessarily would mean a reduction in the accuracy of the map. However, if an office method can be substituted in establishing a part of the control and still maintain the same standards of accuracy, the cost of the map would be lowered. Two office methods now used to establish secondary control are "multiplex bridging" and "radial plotting." Operators of the multiplex do not try to bridge more than twelve photographs. By radial plotting one can do better if perfectly vertical photographs are available. The basic principle to multiplex bridging and radial plotting is the same. The multiplex is an optical method involving three dimensions. Radial plotting is a mechanical method involving two dimensions; the third dimension of relief is projected orthographically on to the plane of the photograph. An advantage of the multiplex is that, in bridging forward, the entire overlap can be seen at once and it is as if an infinite number of pass points were used. However, the greatest advantage of multiplex bridging, as against radial plotting, is that it permits the elimination of tilt. Disadvantages of multiplex bridging are: (1) The final images as viewed have passed through three sets of lenses—the aerial camera lens, the diapositive camera lens, and the multiplex projector lens; and (2) only one flight can be scanned at a time.

In radial plotting, as many adjoining flights can be used as the operator can handle. Adjoining flights help to tighten up the network of secondary control. However, the always present accidental tilt of vertical photographs limits the work of radial plotting.

If the accidental tilt of the photograph were known, it would be possible to use a rectified photograph in the radial plot with the radial center as the nadir point. This would increase the accuracy and span of the radial plot many fold. The usual methods of determining the tilt of a nearly vertical photograph require at least three ground-control points to each photograph. The necessity for this ground control defeats the purpose of expanding secondary control. However, there is a method of determining the tilt of a nearly vertical photograph without the aid of ground control. The answer is "tri-metrogon photography."

Tri-metrogon photography presently will be used for reconnaissance mapping. It is used in construction of planimetric maps of scales of from 1 : 250,000 to 1 : 1,000,000. Tri-metrogon system consists of exposures made with three cameras, one camera facing vertically downward and two cameras turned away

[7] Asst. Civ. Engr., TVA, Chattanooga, Tenn.

[7a] Received by the Secretary July 12, 1945.

from the vertical, facing perpendicular to the line of flight. The oblique cameras are faced so that the horizon and a small portion of the vertical are photographed. All three cameras are exposed simultaneously, photographing a strip of ground from horizon to horizon perpendicular to the line of flight.

Inasmuch as all three pictures of a tri-metrogon series are taken from the same photo station they have the same nadir point. The tilt and thus the nadir point can be determined from the oblique photographs because the horizon is visible. By a simple raying-in process the nadir point can be determined on the vertical photograph. A check is possible inasmuch as two independent determinations of the nadir point can be made from the two obliques. The nadir point of the central photograph having been determined, the tilt can be found easily and the photograph rectified.

Therefore, by taking oblique photographs, together with the verticals just for the purpose of determining the tilt of the vertical, the operator can determine the degree of tilt without ground control. The flying should be done in the usual manner, with the vertical flights overlapping 40%. The additional cost of photography is negligible. Flying cost, in a mapping project, is a small part of the total cost, and it is just as easy to take three pictures at one time as it is to take one. The additional cost of films, processing, and amortization of two added cameras will be slight. Of course, there is the added cost of constructing the radial plot, but this will be greatly overbalanced by the saving in the cost of the horizontal field control that can be eliminated. Another saving would be in the elimination of scaling in the multiplex work, for the radial plot will establish many secondary control points to each model. It may also mean a greater use of the K.E.K. Plotter, a cheaper apparatus that requires control on each model.

CHARLES H. DAVEY,[8] ESQ.[8a]—Mr. Eliel's interesting technical paper is timely and should serve to focus attention on a development in the engineering and scientific field which, from day to day, is finding wider application. In general, the presentation can be accepted without debate, although in detail it is believed that the author's extended knowledge of the stereoplanigraph method has resulted in conclusions that cannot be fully accepted by those who are thoroughly familiar with other methods.

The stereoplanigraph is acknowledged widely among photogrammatrists to be a versatile and probably the most accurate plotting instrument yet devised. It can be accepted as a standard of comparison, if the comparisons are restricted to demonstrated values. Some of the author's statements bear the implication of demonstrated facts, whereas, actually, they partake more of the nature of personal opinions or theoretical considerations that differ materially from practical applications. For instance, under "Comparison of Operation Costs," item (2), citing the relative flying altitude required for each method, in much of the country that is best adapted for mapping by photogrammetric methods and for contour intervals of 50 ft or more, both methods require photographs taken at altitudes that exceed the practical operating

[8] Chf., Section of Photo-Mapping, Geological Survey, U. S. Dept. of the Interior, Washington, D. C.
[8a] Received by the Secretary August 1, 1945.

ceiling of commercially available planes and here the theoretical advantage of the stereoplanigraph over the multiplex method disappears at once. It is here that one outstanding advantage of the multiplex method comes into play as it is possible to set up, at one time, a complete stereoscopic model of a strip exceeding 30 miles in length, at a scale of 4 in. equals 1 mile, and adjust this model to horizontal control as a unit. This is an advantage possessed by no other method and, in practical application, it upsets much of the theoretical advantage as to control required, cited by Mr. Eliel, for the stereoplanigraph.

In item (4), "Plotting the Map," Mr. Eliel states as a fact that a map can be plotted by stereoplanigraph from five to ten times as fast as by multiplex. He has not demonstrated this statement by any logic and it is extremely doubtful if any impartial test has been made under comparable conditions from which a factual comparison could be made. It is most debatable if impartial observers would concede the argument that it is easier and faster to perform the drawing of a map by controlling one's motions through the use of hand wheels operating in the $X$-direction and $Y$-direction than to control them directly by moving the plotting pencil at will. It is further stated in the same paragraph that the multiplex operator will have to draw from two to three times the number of linear inches of contour that the stereoplanigraph operator will have to draw. The writer is not too familiar with operations of the stereoplanigraph, but using Mr. Eliel's data the comparison is about as follows under the comparable condition of mapping with a 20-ft contour interval: The scale of the photographs would be approximately 1 : 27,000 for the multiplex and 1 : 50,000 for the stereoplanigraph; the stereoscopic model would be viewed at a scale of 1 : 10,000 for the multiplex and 1 : 5,000 or 1 : 10,000 for the stereoplanigraph; and the map would be drawn at a scale of 1 : 10,000 and 1 : 8,330, respectively. Thus it would appear that the drawing by the multiplex is less than by the stereoplanigraph. The critical point to consider is that the stereoscopic model is scanned by the operator at the same scale in both instances and for work performed with the same care and attention to detail there appears to be little advantage of one method over the other aside from the admitted advantage of the better optical model which is present in the stereoplanigraph.

It is regretted that definite costs are not available for a careful analysis and comparison of the two methods, for it is the opinion of the writer that each method would be found to have its own particular sphere, where it would operate to best advantage, with perhaps a broad middle ground where there would be little choice between the two. It follows, therefore, that in any extensive project it would be most advantageous to have both types of equipment available.

Mr. Eliel may convey a slightly misleading picture in his statement in the "Summary": "Today in the United States it is probable that nearly all topographic mapping is being done by photogrammetric methods." It is true that photogrammetric methods have a part in nearly all mapping operations at present and in most cases play a major rôle; but the plane table is by no means obsolete. The Geological Survey has maintained an open mind regarding the relative merits of the plane table and photogrammetric methods and it is

believed that, for most economical operation, the best features of both methods should be utilized as conditions dictate, either singly or in combination. It has been demonstrated amply that there are extensive areas in the United States where at the present stage of the art photogrammetry alone cannot compete successfully with the plane table used in combination with photogrammetry.

ROBERT H. RANDALL,[9] M. AM. SOC. C. E.[9a]—Mr. Eliel's paper fills a very real need. His clear description of the principal photogrammetric methods used in United States mapping operations is of value not only to the engineering profession as map makers—for most of the practicing photogrammetrists are engineers—but also to those in the profession who are users of maps.

Although Mr. Eliel's paper is valuable as a clear description of photogrammetric processes, its principal contribution is in the comparative analysis it affords of the economy of these different procedures. The author's conclusions are presented as the result of analyses based upon comparison of the innate characteristics of the various types of equipment, supplemented and reinforced by a wide acquaintance with the various methods in actual practice. For those who, like him, are concerned with the progress of photogrammetry, it may be interesting to note that there is a large and growing volume of cost information by which the economy of the various procedures used, now or in the future, can also be gaged.

All topographic quadrangle mapping now done by or for the United States Government is done to standard specifications. (Mr. Eliel summarizes these specifications, in respect to vertical accuracy, when he states that 90% of the elevations tested must be correct within one half the contour interval.) Further, federal topographic mapping is tested for compliance with specifications, and any federal agency producing and testing such a map is entitled to certify its accuracy by the following statement printed in the margin of the reproduced copy: "This map complies with the national standard map accuracy requirements."

Federal map-making agencies keep cost and performance records of the principal operations involved in topographic mapping, according to a uniform system which they devised jointly at the suggestion of the Bureau of the Budget. Records of work done by contract for federal agencies (as, for instance, the so-called strategic mapping program financed by the War Department but actually performed by five federal mapping agencies and by commercial contractors as well) are also available.

From records such as the foregoing the effectiveness of various photogrammetric methods and combinations of methods may be appraised. In addition to information on work done by the stereoplanigraph, multiplex, and Brock systems, information is accumulating in respect to work done by other methods. Among these may be mentioned the Forest Service method, which employs the K.E.K. Plotter; the stereoscopic plotting equipment of the Coast and Geodetic Survey, which employs photographs taken by a nine-lense camera;

[9] Chf. Examiner, U. S. Bureau of the Budget, Washington, D. C.

[9a] Received by the Secretary August 6, 1945.

and, of course, combination methods involving the use of aerial photographs and plane-table operations.

Mr. Eliel's closing remarks, to the effect that photogrammetry has reached the age of full maturity, are very much to the point. Perhaps it could also be said that photogrammetry, in its present stage, is like a man in the full possession of his powers who is fortunate enough to be situated in virgin territory, with all the opportunities of pioneer enterprise before him; for, although photogrammetry has unquestionably "arrived," further developments are sure to increase its potentialities. New opportunities for its use are constantly arising, and new and improved equipment is being tested. In the matter of control, for example, radar, or some procedure related to it, may greatly improve and cheapen the process of locating the plane at the instant a photograph is being taken; or, to some degree, radar may replace present methods of locating ground points.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# COMPUTING COAGULANT REQUIREMENTS IN SLUDGE CONDITIONING

## Discussion

BY D. L. MCLEAN, AND K. FRASCHINA

D. L. MCLEAN,[18] ESQ.[18a]—The liquid wastes from a modern community vary from day to day and from season to season to such an extent that any mathematical analysis of the amount of coagulant required to condition the sludge from these wastes for partial dewatering by vacuum filters would appear to be a most difficult, if not impossible, task. However, Mr. Genter simply divided the constituents of the sewage sludge into its liquid and solid parts and used the alkalinity of the liquid portion and the volatile matter of the solid portion as his fundamentals. In this most ingenious manner he has reduced the whole intricate problem to the simple mathematical relationship of Eq. 6, which can be applied to all types of sludge and adapted to meet any set of conditions.

Eq. 5, and particularly the nomograph of this equation (Fig. 4), are of particular value and use in the operation of a treatment plant where vacuum filters are used for the partial dewatering of sludge. To check the application of the use of this nomograph and equation, and to show the relationship between the amounts of coagulant determined from them as compared with the actual amount used in practice, the writer offers some data from the treatment plant records of the Greater Winnipeg Sanitary District at Winnipeg. In this plant vacuum filters are used to partly dewater the elutriated digested sludge with ferric chloride as the coagulant. First, a check will be made on the yearly quantities of ferric chloride used for conditioning elutriated sludge; next, a special run will be checked where digested sludge was filtered without elutriation; and finally the filtering runs for a month will be compared, run by run, as well as the monthly average. If the actual results and the nomograph check within reasonable limits, then the value of the nomograph for checking this plant's coagulant requirements will be demonstrated.

NOTE.—This paper by A. L. Genter was published in March, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1945, by Paul D. McNamee.

[18] Superintendent, Greater Winnipeg San. Dist., Winnipeg, Man., Canada.

[18a] Received by the Secretary June 25, 1945.

The results of applying Eq. 5 can be compared with the actual average values for 1944. In that year, 1,906,255 lb of dry filter cake solids were conditioned with 53,739 lb of ferric chloride. This gave an average of 2.8% of ferric chloride used. Properties of the elutriated sludge were as follows: Average alkalinity, 940 ppm; sludge solids, 9%; sludge moisture, 91%; volatile matter, 52%; and ash, 48%. With these values, Eq. 5 yields the following values for the percentage of ferric chloride required: For the alkalinity demand, $\frac{91}{9} \times 0.094 \times 1.08 = 1.03\%$; and, for the solids demand, $\frac{52}{48} \times 1.6 = 1.73\%$. The ferric chloride required for the total dose is the sum of the two, or $1.03 + 1.73 = 2.76\%$. For all practical purposes, this calculated dose is the same as the 2.8% actually used.

Using 940 ppm for alkalinity, as the average for the past three years of operations and combining this with the tabulated averages of 90.5% water, 9.5% solids, 50.7% volatile matter, and 49.3% ash, the calculated dose of ferric chloride is found to be 2.7%, which checks with the 2.7% actually used.

To compare the results by Eq. 5 with those obtained from a special filtering of digested sludge that was not elutriated, the following data may be used:

On an isolated special plant run of digested sludge which was not elutriated, more than 9% of ferric chloride was required to render the sludge filterable. This sludge had an alkalinity of 4,430 ppm, a water content of 93.8%, a solid content of 6.2%, and a volatile content of 57%, with 43% ash. The ferric chloride requirement calculated by Eq. 5 is: For liquid demand, $\frac{93.8 \times 0.443 \times 1.08}{6.2} = 7.2\%$; and, for the solid demand, $\frac{57}{43} \times 1.6 = 2.1\%$. The ferric chloride total requirement is $7.2 + 2.1 = 9.3\%$, which checks with that actually found by the test.

TABLE 5.—COMPARISON OF ACTUAL AND PREDICTED FERRIC CHLORIDE REQUIREMENTS

| September, 1944 | Alkalinity (ppm) | Water (%) | Volatile (%) | Ferric Chloride (%) | |
|---|---|---|---|---|---|
| | | | | Actual | Fig. 4 |
| 2 | 920 | 88.1 | 51.7 | 2.3 | 2.4 |
| 5 | 730 | 90.9 | 52.7 | 2.7 | 2.8 |
| 9 | 830 | 90.8 | 53.8 | 2.7 | 2.8 |
| 11 | 860 | 88.1 | 52.9 | 2.5 | 2.5 |
| 13 | 850 | 88.5 | 49.3 | 2.5 | 2.3 |
| 16 | 940 | 88.4 | 48.2 | 2.1 | 2.2 |
| 18 | 850 | 88.3 | 51.5 | 2.6 | 2.4 |
| 20 | 860 | 90.8 | 51.5 | 2.8 | 2.6 |
| 23 | 930 | 89.9 | 51.7 | 2.9 | 2.6 |
| 26 | 890 | 90.9 | 57.1 | 2.8 | 3.0 |
| 29 | 920 | 91.7 | 56.2 | 2.9 | 3.1 |
| Average | .... | .... | .... | .... | 2.59 |

The results of filter runs for one month, compared run by run with the quantities of ferric chloride taken from Fig. 4, are given in Table 5. The nomograph average for the month (2.59%) compares with the actual percentage (2.54%) computed from the 5,101 lb of ferric chloride used in conditioning 200,912 lb of dry filter cake solids.

These examples of the use of Eq. 5 or Fig. 4 show that for a wide range of conditions it is a most valuable and rapid method to give a check of the ferric chloride used in conditioning sludge in treatment plant operation. A wide divergence between the value given by Fig. 4 and the ferric chloride actually used indicates the probability of inefficiency in some part of the filtering process and should lead to a remedy for any such condition.

As an aid to plant operation Mr. Genter's work is of great value. It should be of equal value to the designing engineer in the selection of the most economical method of sludge disposal. In research it suggests economies to be effected in sewage treatment by concentrating sludges and selecting sources of wash water for efficient elutriation, and emphasizes the value of thorough digestion.

K. FRASCHINA,[19] ESQ.[19a]—The problem of calculating coagulant requirements in sludge conditioning has been reduced by Mr. Genter to a simple mathematical process of apparently wide application which should prove to be very useful to all persons engaged in sludge filtration. In applying his equations to plant practice, however, Mr. Genter warns of the necessity of reliable statistical averages in sampling and analytical methods, and of careful vacuum filter operation.

This discussion will be confined to some theoretical aspects of the equations formulated, and to subsequent operational data for the Richmond-Sunset plant, with tentative explanations of conformity to, and departure from, Mr. Genter's equations.

In the first half of Eqs. 5 and 6 the amount of ferric chloride required to react with the chemically and biochemically contaminated water forming the major portion of the liquid sludge is determined by the reaction of the coagulant with dissolved bicarbonates, as shown in Eqs. (*A*) and (*B*) (see heading, "Sludge-Water Demand for Ferric Chloride"). Any possible effects of phosphates, borates, or silicates, if present, are not considered. It is entirely logical that the reactions given should go practically to completion because the solubility of hydrous ferric oxide is extremely small in water, the reaction proceeding quantitatively even in weakly acid solution.[20] The solubility product of hydrous ferric oxide is not exactly known; it depends somewhat upon the manner of precipitation but is of the order of $10^{-36}$. It follows, therefore, that the amount of iron left in solution at a pH-value of about 4 will be in the order of 0.05 mg per liter. Since alkalinity is also determined, by titration with acid, to a definite pH-value of about 4, it can be inferred, if the foregoing is correct, that the pH-value of the conditioned sludge should be about 4 at the completion of the reactions expressed by Eqs. (*A*) and (*B*), and before any coagulant is available for the "solids demand" of the second part of Eqs. 5 and 6. This is not necessarily the case, at least with elutriated digested sludge. At the Richmond-Sunset plant, for instance, some residual titratable alkalinity remains, for sludge need not be conditioned with ferric

[19] Supt., Richmond-Sunset Sewage Treatment Plant, San Francisco, Calif.

[19a] Received by the Secretary August 1, 1945.

[20] "Textbook of Quantitative Analysis," by I. M. Kolthoff and E. B. Sandell, The Macmillan Co., New York, N. Y., 1936, p. 294.

chloride to a lower pH-value than about 6 for successful filtration. The effect of this on Eq. 6 would be to decrease the coefficient of $Y_1$ and increase the value of $K$.

The second part of Eq. 6 measures the coagulant required for the "solids demand." The fact that the solids-water complex does have a ferric chloride demand can be demonstrated by the presence of considerable amounts of reduced iron in the ferrous state in the filtrate. This "solids demand" is measured in the equation by an empirical constant $K$ and the ratio of percentage volatile matter to percentage ash. Mr. Genter has emphasized the fundamental importance of this ratio in estimating the loss of volatile matter during the digestion of any sludge and the economies that can be realized in digestion and elutriation. These relationships are neatly expressed in Eq. 7. In sludge ignition, organic matter is destroyed and considerable inorganic matter is converted to a variable extent into other forms depending on the temperature of ignition and the particular salts present. The volatile content determined by ignition probably exceeds, and at best only approximates, the actual organic content. The latter presumably determines the proportion of coagulant required; little or no coagulant is required for the mineral matter, as stated by Mr. Genter.

The determination of volatile matter and ash is further complicated by the fact that both are based on the weight of dried solids in the sludge; volatile matter is the percentage loss of solids on ignition, and ash is the percentage remaining. The weight of dry solids is not susceptible of absolute determination. In the standard method,[21] the sample is evaporated overnight in a 103° C oven, cooled in a desiccator, and weighed; or evaporated on a water bath, dried at 103° C, cooled, and weighed to constant weight. At the Richmond-Sunset plant it has been found that a continuous apparent decrease in solids percentage occurs on continued heating at 103° C, probably due to loss of the more volatile organic fractions or to the decomposition of the more unstable organic materials. A limiting or constant weight is approached only after prolonged heating. Furthermore, the dried solids are very hygroscopic; they absorb water very rapidly. Unless weighings are made under such conditions that absorption of water is reduced to negligible proportions, solids and calculated volatile percentages in general will be too high, and computed ash percentage too low. The net effect on the volatile-to-ash ratio depends on the relative loss in weight on heating and gain in weight from moisture absorption, and may either increase or decrease the apparent ratio with corresponding changes in the value of $K$ in Eq. 6. For example, if ash weight remains constant and solids are reported too high by 5%, the apparent volatile-to-ash ratio of a 3% solids sludge of 65% volatile matter will be increased 7.7%; with the same error in a 3% sludge of 50% volatile, the increase will be 10%.

Another factor which may affect the percentage of coagulant required is the rate of flocculation. Hydrous ferric oxide, when first formed, is a positively charged colloid in the acid region to a pH-value of about 6 to 7.5, negatively charged in the alkaline region.[22] In the first case, hydrogen ion and probably

[21] "Standard Methods for Examination of Water and Sewage," published jointly by the A.P.H.A. and the A.W.W.A., New York, N. Y., 8th Ed., 1936, p. 173.

[22] "Handbook of Applied Hydraulics," by C. V. Davis, Editor-in-Chf., McGraw-Hill Book Co., Inc., New York, N. Y., 1942, p. 802.

some ferric ion are adsorbed from solution; in the second case, hydroxide ion and probably some ferric ion are adsorbed. Flocculation is promoted in the acid region by anions and in the alkaline region by cations, the most effective in either case being those of higher valence. The rate of flocculation and, therefore, the amount of coagulant required for any given sludge may depend to a certain extent, then, on the particular dissolved salts present. The rate of flocculation may also be influenced by the presence of certain impurities in the ferric chloride itself, such as calcium, zinc, copper, ferrous iron, hydrogen, sulfate, and phosphate ions.

None of the foregoing invalidates the use of Eq. 6 but some factors have been presented that may serve to explain anomalous behavior in applying Eq. 6 and to indicate modifications in sampling and analytical methods that may be required.

TABLE 6.—COAGULATION REQUIREMENTS; RICHMOND-SUNSET PLANT

| Line | Period | Alkalinity (ppm) | Sludge Assays (%) | | | Ratio[a] Col. 4 / Col. 5 | $FeCl_3$ (%) | | Constant $K$ by Eq. 6 |
|---|---|---|---|---|---|---|---|---|---|
| | | | Solids | Volatile matter | Ash | | Actual | Eq. 5 | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| | (*a*) By Fiscal Years | | | | | | | | |
| 1 | 1941–1942.... | 470 | 3.04 | 65.3 | 34.7 | 1.88 | 4.17 | 4.63 | 1.35 |
| 2 | 1942–1943.... | ...[b] | 2.78 | 68.0 | 32.0 | 2.12 | 5.35 | 5.17 | 1.73 |
| 3 | 1943–1944.... | 465 | 3.00 | 68.4 | 31.6 | 2.16 | 6.05 | 5.08 | 2.04 |
| 4 | 1944–1945.... | 450 | 3.28 | 69.1 | 30.9 | 2.23 | 5.02 | 5.01 | 1.61 |
| 5 | Weighted avgs. | 465 | 3.03 | 67.7 | 32.3 | 2.10 | 5.13 | 4.97 | 1.68 |
| | (*b*) By Months, in the Fiscal Year 1944–1945 | | | | | | | | |
| 1 | July..... | 455 | 3.01 | 69.8 | 30.2 | 2.32 | 5.64 | 5.30 | 1.75 |
| 2 | August..... | 485 | 3.07 | 69.0 | 31.0 | 2.22 | 5.58 | 5.20 | 1.77 |
| 3 | September.... | 505 | 3.44 | 66.8 | 33.2 | 2.02 | 5.95 | 4.76 | 2.18 |
| 4 | October..... | 465 | 3.24 | 68.0 | 32.0 | 2.12 | 4.84 | 4.90 | 1.58 |
| 5 | November.... | 390 | 3.30 | 68.0 | 32.0 | 2.12 | 4.06 | 4.63 | 1.33 |
| 6 | December.... | 425 | 3.63 | 71.2 | 28.8 | 2.47 | 2.94 | 5.18 | 0.70 |
| 7 | January..... | 445 | 3.57 | 70.6 | 29.4 | 2.40 | 3.98 | 5.14 | 1.12 |
| 8 | February..... | 460 | 3.72 | 68.8 | 31.2 | 2.20 | 5.86 | 4.80 | 2.08 |
| 9 | March..... | 420 | 3.77 | 68.8 | 31.2 | 2.21 | 5.18 | 4.69 | 1.82 |
| 10 | April..... | 420 | 3.46 | 69.6 | 30.4 | 2.28 | 4.68 | 4.92 | 1.50 |
| 11 | May..... | 445 | 3.25 | 69.8 | 30.2 | 2.31 | 5.18 | 5.13 | 1.62 |
| 12 | June..... | 465 | 3.09 | 69.9 | 30.1 | 2.32 | 5.64 | 5.29 | 1.75 |

[a] In Table 6(*b*) the values in Col. 6 were computed for each month from the actual accumulated pounds of solids, volatile matter, and ash. The percentages in Cols. 4 and 5 are rounded values computed from these same monthly totals. In some months, therefore, the values in Col. 6 will not coincide with the ratio, Col. 4 : Col. 5. [b] No data available; the value 470 was assumed for purposes of computation.

In Mr. Genter's paper the data presented for the Richmond-Sunset plant are for 1941. Subsequent data computed as weighted averages by fiscal years are given in Table 6(*a*). The data for 1944–1945 are the most reliable statistically because improvements have been made in the measurement of volumes of sludge filtered and in methods of sludge sampling and analyses. In Table 6(*b*), monthly weighted averages during 1944–1945 are given to show the deviations not apparent when weighted averages over the year are considered as in Table 6(*a*).

The data in Table 6 can be considered as valid tests of Eqs. 5 and 6 with the following qualifications: Wartime dislocations in personnel were felt since early in 1942 but were most acute during the fiscal year 1943–1944. The technique of sludge filtration has conformed in most respects with Mr. Genter's criteria of careful operation, although it has been necessary to operate the single filter at practically maximum speed at all times to treat the sludge produced. If more filter area were available, filter speed could be reduced and somewhat greater economy in the use of ferric chloride would be attained.

In Table 6(*a*), except in 1943–1944, fairly close agreement is shown between computed ferric chloride doses by Eq. 5 and actual doses (Cols. 7 and 8); and computed values of $K$ by Eq. 6 (Col. 9) are approximately 1.6, the tentative value set for careful filter operation. The higher values for 1943–1944 can be attributed in part to new and inexperienced operators. This confirms the necessity for competent and experienced personnel if most effective filter operation and maximum operating economy are to be attained.

When the data for 1944–1945 (which, in Table 6(*a*), line 4, show excellent agreement when compared with computed values by Eqs. 5 and 6 on a weighted yearly average basis) are compared by months, rather wide discrepancies appear. In Table 6(*b*), $K$-values computed from Eq. 6 vary from 0.70 to 2.18. The high $K$-value in September may be attributed in part again to changes in personnel, and the high value in February to errors in measurement arising from the small quantity of sludge filtered. The low doses in November, December, and January (Col. 7, Table 6(*b*)) with respective $K$-values of 1.33, 0.70, and 1.12 (Col. 9) are more difficult to explain. At this plant single-stage digestion is employed followed by elutriation of both supernatant and bottom solids from the digester. It is not possible to obtain a supernatant of much less than 1.5% solids with the limited capacity of this single digester. The major portion of elutriated sludge filtered usually consists of solids from the supernatant. From the middle of November to the middle of January the digester was in an abnormal state of activity with most of the sludge accumulated in an actively digesting mass at the top and with only a small amount of partly digested sludge at the bottom. The solids content of the intervening layer decreased to less than 1%. Relatively greater proportions of bottom sludge were withdrawn to elutriation during this period than usual and at the same time there occurred the greatest dilution of elutriating water with storm water. Several possible explanations of the apparently anomalous behavior of this sludge on filtration—that is, less coagulant required than computed—suggest themselves. First, digestion had not proceeded as far as usual and, in the sludge solids, intermediate digestion products including more than usual quantities of scum solids containing such excellent aids as disintegrated hair, paper, etc., were present whose coagulant demand no longer could be evaluated properly by the volatile-to-ash ratio; and, second, the flocculating effect of dissolved salts in the associated water phase was changed by the presence of unusual soluble decomposition products.

The plant was shut down for mechanical repairs from the middle of January to the end of February; the digester then reverted to normal activity. The aforementioned atypical behavior in sludge filtration has not recurred since.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# EFFECTS OF RADIANT HEAT ON REINFORCED-CONCRETE RIGID FRAMES

## Discussion

BY I. OESTERBLOM

I. OESTERBLOM,[10] M. AM. SOC. C. E.[10a]—Had the author used a more restrictive title his paper would have had mathematical interest, limited, however, to the extent that it is based on the conventionally assumed (but approximate) relations between stress and strain. The paper does not seem to cover the subject of radiant heat as applied to frames.

Assuming that radiant heat comes from the sun, the investigator must begin with the basic radiation formula:

$$H = \sigma \epsilon_1 \epsilon_2 dA_1 dA_2 \frac{\cos O_1 \cos O_2}{\pi L^2} (T^4{}_1 - T^4{}_2) \ldots\ldots\ldots\ldots (8)$$

Eq. 8 defines the quantity $H$ that is radiated from $dA_1$ to $dA_2$ with radiating temperatures $T_1$ and $T_2$ (absolute), respectively. The symbols $\epsilon_1$ and $\epsilon_2$ represent the emissivities and $\sigma$ represents the Stefan-Boltzmann constant. The angles of incidence (the symbols $O_1$ and $O_2$) may be set equal to zero for solar radiation, if a perpendicular plane is assumed at the earth.

With what is known about dimensions, distances, and temperatures, Eq. 8 gives a theoretical maximum value of the heat radiation on earth of about 400 Btu per sq ft per hr for any perpendicularly exposed area of black body emissivity—when no atmospheric interception occurs before the rays of the sun reach this area. Usually only 300 Btu is taken as the reduced maximum.

Some of this heat is absorbed, some is re-radiated into cosmic space to −50° F, and some is re-radiated into the atmosphere. When the atmosphere is heavy only a part of the 300 Btu can reach the earth. Much of that which arrives is re-radiated to the heavily absorbent atmospheric moisture.

Then there is also the angle of incidence which, for construction planes, generally is not perpendicular to the rays of the sun. Here again the effect is very much reduced through the low value of $O_2$. In the middle of the day

NOTE.—This paper by Milan A. Johnston was published in June, 1945, *Proceedings*.

[10] Engr., Carbide & Carbon Chemical Corp., South Charleston, W. Va.

[10a] Received by the Secretary July 13, 1945.

only roofs are heavily affected; and walls are very lightly affected. During the forenoons and afternoons even the walls are spared because the atmospheric penetration distance is so much extended. The over-all, all-day, picture thus is not a bad one.

The temperature of the air adjacent to any exposed area is now a function of the residual heat which is not re-radiated. It is not a function of that alone, however. The heat that has penetrated the atmosphere on its way from the sun must equal that which has been re-radiated into space, plus another portion which the construction is able to absorb and store or pass through to the interior. The second part must pass through the all-important surface film, a concentration of air molecules with definite thickness and temperature gradient which depends on the thermal balance and the convection velocity for its magnitude.

Even assuming that the residual heat is known, the establishment of a balance between the film and the interior is very difficult. Mass, and the shape of the construction, affect both heat storage and conduction, according to the Fourier laws, and the film is governed by the radiation and convection laws. Thus one must resort to guesses about film temperatures; but this can be done only if, afterward, one proves that the guesses were reasonably correct and that they actually apply to both film and construction.

The author has assumed surface temperatures as reported by F. B. Rowley and C. E. Lund of the University of Minnesota,[6] Minneapolis, Minn. One would like to know how high those temperatures were, how they varied during the day, how the surface was located, and what mass and form were below the exposed surface. This information would be useful not only to follow the author's reasoning, but also to enable an engineer to repeat the method for other cases, if it seems good enough. It is quite possible that, in the light of the foregoing argument, the "comparable surface" (see heading, "The Effect of Radiant Heat") computed in the "Numerical Example" is far different from the one that the author asks his readers to accept on faith. The proposed "comparable surface" is quite insufficient to establish reasonable similarity.

In setting up his numerical example, the reason for selecting a slab is not clear. In a thermal sense a frame element is not a slab, and in Fig. 2 it is not shown as such. Reference to the work of R. W. Carlson,[7] Assoc. M. Am. Soc. C. E., and his (no doubt) very specialized and seemingly graphical solution is also questionable. It would be almost impossible to extend any graphical solution to a two-way, or a three-way, flow of heat, and that is the type of problem encountered in a frame element protected by walls. For an exposed frame, involving a four-way flow of heat, the possibility of extending a graphical solution is even more remote.

Ever since J. B. Fourier[11] first raised the question of flow and absorption of heat this problem has been treated extensively by various authorities, gen-

[6] "Heat Transmission Through Insulation as Affected by Orientation of Wall," by Frank B. Rowley and C. E. Lund, *Technical Paper No. 44*, Univ. of Minnesota Eng. Experiment Station, Minneapolis, Minn., 1943.

[7] "A Simple Method for the Computation of Temperatures in Concrete Structures," by Roy W. Carlson, *Journal*, A.C.I., November–December, 1937, pp. 89–102.

[11] "Théorie du mouvement de la chaleur dans les corps solides," by J. B. Fourier, l'Académie Française, 1812.

erally physicists and mathematicians (Laplace, Poisson, Green, Lamé, Heine, Liouville, Thompson (Lord Kelvin), and Riemann) and more recently also by engineers (Gurney, Heilman, King, McAdams, McMillan, Newman, Preston, Schack, Schmidt, and others). However, Fourier's theory does not take into account heat used for evaporation of water—free, adsorbed, or crystallic—which is present in most construction materials. There is a very definite and well-established procedure, therefore, by which to compute the thermal changes in bodies of various shapes.

A homogeneous section must be assumed (the problem is exceedingly difficult even so) and, therefore, reinforcement cannot be included in the setup. However, this fact is not important. The conductivity of steel is so high in relation to the heat-storage capacity that the disturbing influence through a very high diffusivity acting on the small area of reinforcement is insignificant.

The Fourier equation for heat flow through a prism—with the use of conventional symbols and the coordinate axes at one of the corners—reads as follows:

$$\alpha \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = \frac{\partial u}{\partial \tau} \qquad (9)$$

in which the thermal diffusivity is given by $\alpha$, the same as in the paper. Symbol $\tau$ denotes time, and

$$u = \frac{t_1 - t}{t_1 - t_o} \qquad (10)$$

is the temperature function, which expresses how far, in temperature increase, the point $x,y$ has gone as compared to the ultimate possible change, when the uniform mass at temperature $t_o$ has reached the surface temperature $t_1$ at that point.

The Fourier equation (Eq. 9) is for a two-way (or a four-way) flow of heat along the $x$-axis and the $y$-axis—in other words it applies to a rectangular prism only. By dropping one of the axes and making the equation apply to a slab, the solution becomes greatly simplified.

One can thus write, in trigonometric form:

$$S_x(\theta_x) = \frac{4}{\pi} \left[ e^{-(0.5x)^2\theta} \sin \frac{\pi x}{2 b_x} + \frac{1}{3} e^{-9(0.5x)^2\theta} \sin \frac{3 \pi x}{2 b_x} + \frac{1}{5} e^{-25(0.5x)^2\theta} \sin \frac{5 \pi x}{2 b_x} + \frac{1}{7} e^{-49(0.5x)^2\theta} \sin \frac{7 \pi x}{2 b_x} + \cdots \right] \qquad (11)$$

Values of $S_x(\theta_x)$ from Eq. 11 may be tabulated and charted so as to obtain a quick solution of specific problems. The symbol $\theta_x$ is defined as:

$$\theta_x = \frac{\alpha \tau}{b^2{}_x} \qquad (12)$$

in which $b_x$ is the distance from the outer surface to the thermal center of the

slab; and, as in Eq. 10:

$$S_x(\theta_x) = \frac{t_1 - t}{t_1 - t_o} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

The solution has been presented by many authorities following parallel methods. An outline derivation of Eq. 11 is offered by T. K. Sherwood and C. E. Reed[12] who in turn refer to H. S. Carslaw[13] for further details. The derivation of Eq. 11 has been presented by F. C. W. Olson and O. T. Schultz. All writers make a general acknowledgment to other helpful authorities.

For reintroducing the $y$-axis and finding the following simple solution for a two-way heat flow—

$$S_x(\theta_x)\, S_y(\theta_y) = u \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

—credit is due especially to Albert B. Newman[14] who presented a proof that Eq. 14 is true. Thus, by taking two infinite slabs perpendicular to one another and finding $S(\theta)$ for each at a common point $(x,y)$, the investigator is able to establish first the temperature function $u$ (which is common to the two) and then the temperature for the point and, at the time, by solving Eq. 10 for the temperature $t$:

$$t = t_1 - u\,(t_1 - t_o) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Various graphs for the determination of $S(\theta)$ have been produced by H. P. Gurney and J. Lurey[15] although the logic supporting them has never been published. By the use of such special graphs the problem of a rectangular prism thus becomes quite simple: (1) Establish the relative location of the point $x,y$ in terms of $b_x$ and $b_y$; (2) indicate the diffusivity by $\alpha = \frac{k}{c\,\rho}$; (3) find the two values of $\theta$ by Eq. 12; (4) find the values of $S(\theta)$ by the charts; (5) combine the two values of $S(\theta)$ as shown by Eq. 14; and (6) find the temperature of the point by Eq. 15.

The complication introduced by variable surface temperatures now enters, and a problem already difficult (to find a rational solution such as Eq. 11) becomes almost impossible. The variable boundary conditions would make it impossible, and even if that were not so the variability cannot be expressed directly and rationally.

However, it is possible to assume that the average surface temperature is constant for the duration, and to compute the interior temperatures on the basis of this approximation. One would then be ready for the structural problem of stress, as soon as enough critical points had been computed. The effect of the thermal stress would be equivalent to a supplementary axial eccentric load.

---

[12] "Applied Mathematics in Chemical Engineering," by T. K. Sherwood and C. E. Reed, McGraw-Hill Book Co., Inc., New York, N. Y., 1939.

[13] "Mathematical Theory of the Conduction of Heat," by H. S. Carslaw, The Macmillan Co., London, 1921.

[14] "Heating and Cooling Rectangular and Cylindrical Solids," by Albert B. Newman, *Industrial and Engineering Chemistry*, May, 1936, p. 546.

[15] *Industrial and Engineering Chemistry*, 1923, p. 1170.

For a section away from the walls one would have the top exposed and subject to high temperature. For a section next to a wall the top and one side would have higher temperatures than the inside faces. In all cases there would be set up an extra stress covering the entire area and not (as shown in Fig. 5) applied above the neutral axis only.

Fig. 5 is based on conventional approximations regarding stress and strain, and that is scarcely permissible when one ventures into a new field. The true answer must be ascertained first, and only then may one be allowed to look for reasonable simplifications. Serious consequences have followed opposite procedure. Did the London Bridge fail in 1832 because of the overextension of a method? Most likely it did not, because, when it was built, one did not know even the basic method; but the second Quebec Bridge fell—due to a fateful extrapolation of a limited formula—in 1916. Because of his misapplication of the basic thermal theory, as well as the original stress diagram due to load, the author should recompute his example so that comparable figures would be available as an illustration. Only the author can do this, on the basis of the assumed surface temperatures used in the example, but not stated in the paper. The stresses thus computed certainly would be substantially less than those reported in the paper. Indeed, if it were not for this probability there would have been many failures of many famous structures due to thermal stress.

The integration of stress into deflections and moments is elementary, and many methods might be used for this purpose. The revision proposed in these notes will cause a surprise, however. A frame next to an outer wall would show lateral deflection and, as a result of that deflection, torsional shears would occur next to the supports. A point that is often overlooked should be given thought in this connection: One cannot assume an average temperature, arbitrarily, as a basis for computing temperature stresses. The basic temperature of the frame (the atmospheric temperature plus something added for chemical heat) is the one at which concrete takes its initial set when poured. The chemical heat is a relatively small item on ordinary frames but a heavy item on heavy construction. Cold weather construction thus gets a heavy, extra load during the summer and hot weather construction is subject to a considerable reduction.

The writer would like to summarize his criticism as follows: "David" went out to slay a "Goliath," but he did not search first for a slingshot, and thus this "Goliath" is still very much alive. Nevertheless, the entire profession should be highly grateful for the paper, because the author has ventured into highly dangerous and difficult territory which should have been explored extensively but which all research engineers have feared to enter. He may indeed have stirred into action many thoughtful but hesitant engineers who have constructive ideas to present.

It is not solar radiation alone that should be considered; neither is the concrete frame the most important structure. There are other sources of heat; and there is also fire, accidental fire. When one gives thought to this vast field it comes to mind at once that so little has been done to produce a safe design. Why? Because so little is known, or expressed in rational form.

This is true even in the important field of fireproofing. Suppose that a 3-in. fireproofing cover is safe for a given steel column. The same covering may be applied to another section where similar heat intensity is expected; and the column fails nevertheless because of a fanciful curiosity of shape and texture. Why? Simply because the diffusivities of compounded materials in relation to the areas they cover, and the flow paths they present, offer unlimited combinations of time and temperature, for which no one has ever found a rational formula.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# INTERRELATION OF CERTAIN STRUCTURAL CONCEPTS

## Discussion

BY T. F. HICKERSON

T. F. HICKERSON,[30] M. AM. SOC. C. E.[30a]—This is an interesting and valuable paper; but these structural concepts and the interrelated exact carry-over factors do not constitute an integral part of the Cross method of moment distribution, as the casual reader might infer from the author's introduction.

To the list of algebraic relations in Table 1, one more column might be added for completeness, as indicated in Table 5. Here certain concepts are expressed in terms of "carry-over factors." The term "degree of restraint" as introduced by the author might well be omitted. It serves no useful purpose except as an approximate check on the "degree of fixity"; the $K$-values alone could serve the same purpose in a much simpler manner. Thus, in Fig. 3:

$$F_{BA} = \frac{K_{BJ} + K_{BC} + K_{BH}}{K_{BJ} + K_{BC} + K_{BH} + K_{BA}} \quad \ldots\ldots\ldots\ldots (82)$$

approximately.

The letter symbols in Appendix I appear, in the main, to be well chosen. Subscripts must be added to avoid confusion. The idea of nodal intercept tends to confuse rather than clarify the concept of "carry-over factor," and should be omitted from the definition. Instead of defining $K\ (= I/L)$ as "stiffness ratio," the writer prefers "stiffness coefficient" or "nominal stiffness" inasmuch as the real stiffness (or restraint) of a structural member is dependent not only on its $K$-value but also on the degree of fixity at the far end of the member.

A symbol should be added for "moment split." This symbol indicates how a moment that is carried over to the end of a member is to be balanced into, or absorbed by, each of the other members at the joint. Evidently this induced moment divides itself among the receiving members in accordance

NOTE.—This paper by Camillo Weiss was published in January, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1945, by Ralph W. Stewart, and R. C. Brumfield; June, 1945, by Max W. Strauss; and September, 1945, by Leon Beskin, Alexander Dodge, and William C. Spiker.

[30] Prof. of Applied Math., Univ. of North Carolina, Chapel Hill, N. C.

[30a] Received by the Secretary August 17, 1945.

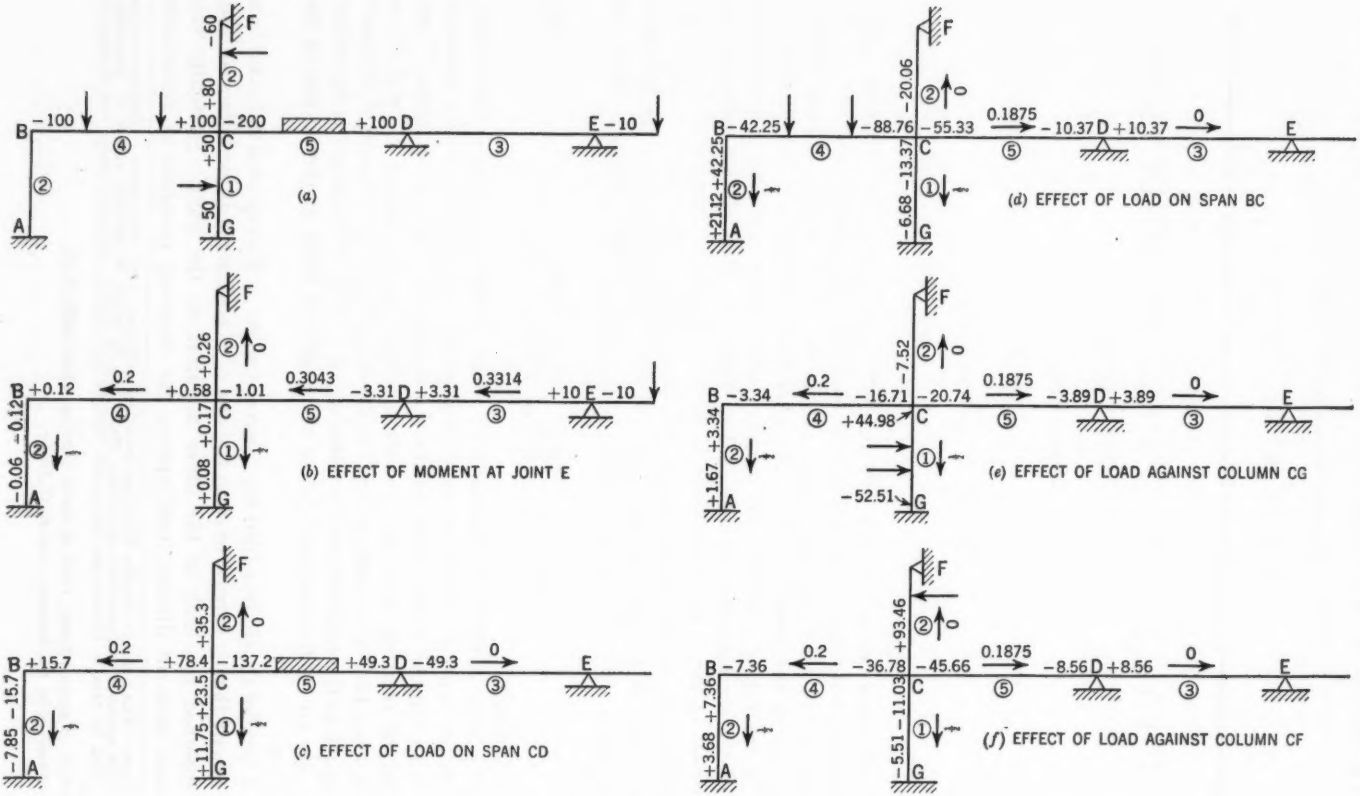

(a)
(b) EFFECT OF MOMENT AT JOINT E
(c) EFFECT OF LOAD ON SPAN CD
(d) EFFECT OF LOAD ON SPAN BC
(e) EFFECT OF LOAD AGAINST COLUMN CG
(f) EFFECT OF LOAD AGAINST COLUMN CF

FIG. 10.

with their relative stiffness values. This "moment split" ratio is likely to be confused with the "distribution factor." As a rule, the writer prefers not even to compute the distribution factor; that is, instead of assuming the "near end" fixed—a condition that does not actually exist—just use the real degrees of

TABLE 5.—STRUCTURAL CONCEPTS IN TERMS OF THE CARRY-OVER FACTOR

| Structural concept | Symbol | Carry-over factor |
|---|---|---|
| Nodal intercept ratio | $n_{AB}$ | $\frac{1}{f_{AB}+1}$ |
| Distribution factor | $d_{AB}$ | $\frac{1-2f_{BA}}{1-f_{AB}f_{BA}}$ |
| Degree of fixity | $F_{AB}$ | $\frac{3f_{BA}}{2-f_{BA}}$ |
| Restraint factor (or stiffness factor) | $R_{AB}$ | $\frac{6f_{BA}K_{AB}}{1-2f_{BA}}$ |

fixity at each end of the loaded member as arguments (in a table compiled by the writer[31]) and write down the coefficients giving final moments in the member. These end moments are then balanced into the restraining members, and carried on to remote joints in greatly reduced quantities.

Instead of Eqs. 39, a parallel set is as follows:

$$M_{BA} = [\text{FEM}_{BA} + \text{FEM}_{AB}\tfrac{1}{2}(1 - F_{AB})](1 - d_{BA}) \ldots\ldots (83a)$$

and

$$M_{AB} = [\text{FEM}_{AB} + \text{FEM}_{BA}\tfrac{1}{2}(1 - F_{BA})](1 - d_{AB}) \ldots\ldots (83b)$$

The only advantage that can be claimed for Eqs. 83 in comparison to Eqs. 39 is that they contain two variables, in addition to the fixed-end moments, instead of three variables.

Instead of Eqs. 83 the writer, as a rule, prefers to use the following:

$$M_{AB} = \frac{4F_{AB}H_{AB}}{D} \ldots\ldots (84a)$$

$$M_{BA} = \frac{4F_{BA}H_{BA}}{D} \ldots\ldots (84b)$$

in which: $H_{AB} = \text{FEM}_{AB} + \frac{1}{2}\text{FEM}_{BA}(1 - F_{BA})$; $D = 4 - (1 - F_{AB})(1 - F_{BA})$; and $H_{BA} = \text{FEM}_{BA} + \frac{1}{2}\text{FEM}_{AB}(1 - F_{AB})$.

If either end is fixed, $D = 4$, and Eqs. 84 are greatly simplified; but the real advantage of Eqs. 84 is their adaptability to a table of coefficients, which has been compiled.[31] Thus, with $F_{AB}$ and $F_{BA}$ as arguments, the moments at the ends of the loaded member can be written down most easily.

To illustrate the foregoing the writer has solved the frame shown in Fig. 10(*a*). The results are written on the members in Figs. 10(*b*) to 10(*f*). This method of solution serves also as an influence diagram indicating which combination of loads will produce maximum effects. If all the members are

[31] "Statically Indeterminate Frameworks," by Thomas F. Hickerson, Univ. of North Carolina Press, Chapel Hill, 1937, pp. 90–93.

loaded simultaneously, the total moments are as follows: $M_A = +18.56$; $M_{BA} = M_{BC} = +37.13$; $M_{CG} = +44.25$; $M_{CB} = +114.25$; $M_{CF} = +101.44$; $M_{CD} = -259.94$; $M_{DC} = -M_{DE} = +23.17$; $M_{ED} = -M_{DE} = +10$; and $M_G = -52.87$.

The fact that the "degree of restraint" and "carry-over factors" for interior spans of continuous beams of constant moment of inertia and equal spans converge toward a fixed quantity was discovered by the writer many years ago. Thus the "degree of restraint" produced by three or more equal spans is 46% whether the far terminal be hinged or fixed.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# ELIMINATION OF CROSS CONNECTIONS IN LOS ANGELES, CALIF.

## Discussion

BY ROBERT P. LOWE

ROBERT P. LOWE,[10] ASSOC. M. AM. SOC. C. E.[10a]—The difficulties that must be surmounted to produce an effective program for elimination of cross connections are given in this paper. The formation of the Cross-Connection Control Advisory Committee is one solution to the diversity of interested jurisdictions which have hindered vigorous action against this recognized danger to the public health. Therefore, the ability of the City of Los Angeles to combine the jurisdictions of the Building and Safety Department, the Health Department, the Water and Power Department, and the Board of Mechanical Engineers for this specific purpose is commendable. A number of cities have made extensive surveys, but nearly all have resulted in very little actual elimination on a large scale, because of complications in authority which were not entirely overcome. This condition will continue to exist unless more authority can be granted under one jurisdiction, as the Los Angeles solution is not always possible.

It is obvious that considerable educational work is necessary to indicate the urgency for the elimination of the cross-connection menace which is often not realized by the layman nor even by the average civil engineer who is not associated with the sanitation field. This was quite evident on a cross-connection elimination survey prosecuted by the War Relocation Authority at its Centers in 1943 and 1944. In many cases plumbers were found to be most aware of the conditions, but they could not control the allocation of time and funds for the changes. Action by the sanitary engineer in the Health Section finally overcame most of this difficulty. Removal was obtained in practically all cases by persistent check ups with renewed emphasis of importance to every one in direct authority.

The condition at the army fort in the Los Angeles Harbor area was no doubt an extreme case for army installations. As the Centers for the War Relocation

NOTE.—This paper by Harry Hayes was published in March, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1945, by Roy O. Van Meter.

[10] San Engr., War Relocation Authority, Washington, D. C.

[10a] Received by the Secretary July 11, 1945.

Authority are built on army camp style, they can be compared very easily with the usual cantonment type of installation for troops. This consists of central block latrines and mess halls that centralize and simplify plumbing installations. Very little trouble was found with these installations. In contrast, the Center hospitals were found to be the source of about 90% of cross connections of all types. A few were found at sewage treatment plants and in repair shops. Strict control of hospital plumbing can eliminate the largest source of possible water supply contamination at most army installations of the cantonment type.

The Public Health Service Drinking Water Standards that were adopted on September 25, 1942, raised the requirements for drinking (and culinary) water provided by common carriers for the use of passengers carried in interstate traffic. These standards recognize the seriousness of the cross-connection menace and emphasize responsibility for potable water throughout the distribution system. In the past, great stress has been placed on the source of supply and purification plant control; therefore it appears that modern water purification has become inconsistent with distribution system control. This fault does not lie with any one group. Instead, it is a jurisdictional problem that is clearly indicated by the formation of the Los Angeles Cross-Connection Control Advisory Committee.

*Conclusion.*—All indications point to the fact that improved distribution control is necessary before present water-purification standards will be consistently beneficial. War, with its necessity for speed and temporary installations, has focused more importance on the seriousness of the cross-connection problem, and there seems to be little doubt that the postwar period will see action on a larger scale. However, it appears that legislation and educational methods on a state-wide basis will be necessary for uniform control, as the jurisdictional difficulty has proved to be slow and almost insurmountable in most instances.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# DESIGN OF ROADBEDS

## Discussion

BY CHARLES W. BRITZIUS, J. T. L. MCNEW, AND GUSTAVO PÉREZ GUERRA

CHARLES W. BRITZIUS,[21] ASSOC. M. AM. SOC. C. E.[21a]—The paper seems somewhat academic. Admittedly, to provide a stable subgrade, it is necessary to keep the moisture content low, and this can be done best by compacting the subgrade soil to a low void content. It is impracticable, however, to compact the heavier soils to such a density and low void content that the critical moisture content would not be exceeded. A compacted soil mass does not absorb water in its air-void spaces without expanding, and thus increasing in volume.

TABLE 7.—FACTORS THAT REDUCE EXPANSION AND WATER ABSORPTION

| Material | Percentage Retained: | | | | Liquid limit (%) | Plastic limit (%) | Plasticity index | Optimum moisture (%) | Maximum density[a] | Air Voids at 95% Proctor Density: | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | No. 10 | No. 270 | 0.005 mm | <0.005 mm | | | | | | Total voids (%) | Moisture content[b] (%) | Plastic limit (%) | Col. 11 / Col. 12 |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
| Sandy loam... | 0 | 63 | 27 | 10 | 23 | 14 | 9 | 10.0 | 127.5 | 26 | 13.4 | 14 | 96 |
| Clay loam.... | 0 | 38 | 42 | 20 | 35.5 | 16.5 | 19 | 15.5 | 111.4 | 35 | 20.4 | 16.5 | 124 |
| Clay..... | 0 | 10 | 35 | 55 | 39.5 | 20.5 | 19 | 19.5 | 99 | 43 | 28.7 | 20.5 | 140 |

[a] Modified Proctor density, in pounds per cubic foot. [b] Moisture content when voids are filled.

The factors that will reduce the amount of expansion and water absorption are the weight of the overlying soil, the moisture available, and the moisture content and density at the time of compaction. The writer wishes to present some data on the latter factor. Consider three soils, a sandy loam, a clay loam, and a clay. Their properties are as given in Cols. 1 to 9, Table 7.

NOTE.—This paper by Ira B. Mullis was published in April, 1945, *Proceedings*.

[21] Twin City Testing & Eng. Laboratory, Inc., St. Paul, Minn.

[21a] Received by the Secretary May 25, 1945.

A reasonable, practical, and workable requirement for compaction of these soils is 95% of the Modified Proctor maximum density. At this density the air voids in the soil mass will be as shown in Cols. 10 to 13, Table 7. The air-void values, for these soils, do not conform to the suggested requirement that the pore space be restricted to such a value that, when the space is filled, the moisture content does not exceed 70% of the plastic limit. Because extreme densification is not practicable, thicker and more carefully graded base courses must be used to carry the loads.

Compacted soil masses swell and absorb water to some extent after compaction, until a so-called equilibrium is reached. Soils compacted drier than the optimum swell and absorb water to such a great extent that they finally reach a high moisture content—the same condition that would have resulted if the soils had been compacted beyond the optimum originally. The new residual density curve for sandy and silty soils lies just below the Proctor moisture density curve; but, as the soil becomes heavier, the top of the curve moves to a higher moisture content. The practical implication is that a soil should be compacted to the specified density at such a moisture content that the lowest final moisture content and greatest density will exist. For most soils this moisture content is just greater than the Proctor optimum, and for expansive and gumbo clays it may be considerably greater.

In field soil compaction, many times it is expedient to compact soils drier than the optimum. To prevent greater swell and moisture absorption than would have occurred if the compaction had been done at the optimum, a greater density should be required.

The observation made by Mr. Mullis (see heading, "Change of State: Consolidated Roadbeds") to the effect that soils compacted to low air voids are resistant to frost action is misleading. Soils are adversely affected by frost action not because of any change in the soil, water, and air constituents of a unit soil mass due to freezing, but because of an increase in moisture content due to the movement of water toward the source of the freezing. If the soil is silty and thus of a type that has a favorable porosity and capillarity, the heaving due to moisture accumulation will be as much as 1 ft. When the frost heave has melted, a frost boil remains.

J. T. L. McNEW,[22] M. AM. SOC. C. E.[22a]—One solution to the problem involved in the design of stable earthwork is proposed by the author. Although not specifically identified as such, the author has upheld vigorously the fundamental engineering principle that diagnosis must be thorough and complete before any proposed solution may have better than an average opportunity for success.

Design of earthwork structures involves, from the very beginning, a frank and straightforward admission that soils are materials far different from steel, concrete, and stone. These latter materials are almost always such that they may be assigned certain average characteristics, and the designs resulting therefrom may be expected to have average behaviors. On the other hand,

[22] Vice-Pres. for Eng., Agri. and Mech. Coll. of Texas, College Station, Tex.

[22a] Received by the Secretary June 28, 1945.

soils are subject to almost all of the "peculiar" behaviors and are influenced in their permanent behavior by many more natural occurrences than is steel.

Probably the most serious oversight now common in the design of soil structures is the failure to modify a "theory" as dependent upon the variation between the climate at the site of the structure and the "climate" of the laboratory. Certainly the roadbed that is successful in arid regions of the southwestern United States might not be successful in the Gulf Coast section, nor in regions subject to hard freezes and thaws. The addition of water beyond a small proportion weakens plastic soils so far as load-carrying ability is concerned, and the water that enters a dry, compacted soil first enters through pore space. Just what happens after pore space in a soil is filled with water is quite another matter. Unless such things are considered before construction is started, and unless provisions are made for them, new failures may, and often do, result.

As a principle of design to be adopted, the author's concept, that consolidation should be such as to attain a chosen percentage of pore space in terms of the lower plastic limit, is unquestionably sound. Such a procedure may provide the best design for a given soil; and, as the author states, such densities may not be attainable with reasonable construction effort. The requirement of high density may properly lead to the use of the author's suggested procedure in the selection of better raw materials, or even to the use of other expedients to exclude water from the irreducible pore spaces. Perhaps the soil structure may be planned for a locality where the intrusion of water seldom occurs.

There seems to be no substitute yet for competent and careful engineering diagnosis of soil characteristics and for an accurate evaluation of the "loads" to be carried, both as a result of climate and traffic, in the engineering design of roadbeds. "Rules of thumb" probably do not exist that will replace the engineer's work in a diagnosis of all the problems prior to the prescription of a remedial treatment or cure. It is stimulating, therefore, that Mr. Mullis has insisted that the fundamental behaviors of soils be studied thoroughly before a soil structure is erected. There are reasons for the inability of a soil to do some things; there are ways in which almost all types of soil can do certain jobs; and there are many ways that these types of soil can be misused. Therefore, the engineer must discover these ways with the aid of his "on the job" investigations of the fundamental soil behaviors and, in the light of these findings, propose the design most likely to succeed. Truly, as the author states (see "Introduction"): "The method of approach is a rational one, the principles of which have been long understood, although habitually overlooked."

GUSTAVO PÉREZ GUERRA,[23] JUN. AM. SOC. C. E.[23a]—Considerable difficulty has been encountered, in several cases, in attempting to compact soils that were at their optimum moisture content. In most such cases the difficulty is attributed to imperfect control of embankment operations. Disregarding that

[23] Civ. Engr., Parsons, Brinckerhoff, Hogan & MacDonald, Caracas, Venezuela.

[23a] Received by the Secretary August 20, 1945.

simple explanation, many engineers have thought that there was a missing link in the practical application of the compaction principle.

The method of design proposed by the author is founded on well-established facts and sound reasoning; it should be considered with interest by soil engineers. To the writer it seems that the case presented is so clear and evident that this paper should lead to a complete revision of compaction standards.

When it was developed in 1933, the Proctor method was a valuable scientific tool which eliminated rule-of-thumb methods. It was later standardized and the mechanics of its routine application obscured several well-known and important factors.

It is a known fact that, more often than otherwise, the optimum moisture content for standard compaction is only a few points below the lower plastic limit. It also is known that there is a "slip" when that optimum content is exceeded. The basic process of compaction was not misunderstood, only that the standard compaction was the best that could be obtained using roller pressures of about 200 lb per sq in., as was the case in the early thirties, while at present it is common to use pressures as high as 290 lb per sq in. for regular work and much higher pressures for special compaction problems. Several manufacturers list, as standard, rollers producing a ground pressure of 390 lb per sq in. The tendency for rolling to create higher ground pressures has emphasized the deficiencies of the standard method and at the same time has made possible the development of the new method of design proposed by the author.

In general, the writer agrees with the conclusions of the paper and draws the following practical conclusions:

(1) The curves of moisture-density relation obtained by the use of different compacting forces can be considered as theoretically tangent to the zero air-voids curve with the ascending maxima located on a curve roughly parallel to that curve. The standard curve determined with a specified number of blows is one of the lowest of the range and thus this test furnishes a minimum of compacted density with a safety factor equal to or slightly greater than unity. It is obvious that the designer should try to avoid that minimum.

(2) Rollers of different sizes are available in the market, and their indiscriminate use in conjunction with the standard compaction test defeats the advantage gained by the use of higher pressures.

(3) Extensive correlated laboratory-field tests are urgently needed to set up new compaction standards. There appears to be no need to discard the popular standard apparatus for routine determinations, provided enough correlation is secured. Nevertheless, the basic determinations of critical moisture for the new approach, and the need for load and weather tests, require the use of the stress-strain method and the testing of full-size sections.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# FLOW FROM DRAINAGE BASINS DETERMINED BY SHORT-TERM RECORDS

## Discussion

BY SAMUEL ROBERTS

SAMUEL ROBERTS,[11] JUN. AM. SOC. C. E.[11a]—The extension of short-term records for design purposes is a problem of frequent occurrence in flood control and drainage practice. Where long-term records are available, a straightforward procedure may be used to present the data in frequency form. The flood peaks are plotted in accordance with a suitable formula, such as Eq. 1, and a curve is drawn to fit all the plotted points, as in Fig. 4. For short-term records this procedure must be modified by an examination of the data to determine whether or not the record is a truly representative sample of the long-term record and, if not, what correction should be applied. The frequency curve finally drawn need not be the average of all the plotted points. If there is a sufficient scatter of points in a short record, such as ten years, it may be expected that one or more of the high peaks should have greater recurrence intervals than ten years, and that the other peaks will define the lower portion of the frequency curve. The final curve drawn with this premise in mind, therefore, may deviate considerably from the plotted positions of the higher peaks.

For plotting purposes, the writer prefers log-log paper instead of the probability paper used by the author and he prefers the plotting of the data by frequency in years, the reciprocal of $F$ in Eq. 1. Peak discharge frequency curves plotted by this method usually are parabolic in shape, making extrapolation comparatively simple. Deviation from this shape occurs only in a watershed displaying material differences in characteristics between moderate and severe floods, such as would be caused by storage or by regulation.

The data given in Tables 2 and 3 have been replotted on log-log paper in Fig. 8, in accordance with the formula:

$$f = \frac{t}{N - 0.5} \quad \ldots\ldots\ldots\

in which $f$ equals the frequency of recurrence in years; $t$ equals the number of years of record; and $N$ equals the order number of the peak (highest equals 1, etc.). The Yarnell frequency curve, as given in Fig. 4, has also been replotted in Fig. 8. It may be noted that the plotted rainfall points agree reasonably well with the Yarnell curve and no adjustment in frequency is warranted. Furthermore, the plotted peak runoff points for the short records show a good scatter in magnitude, indicating a representative sample of the long record in each case.

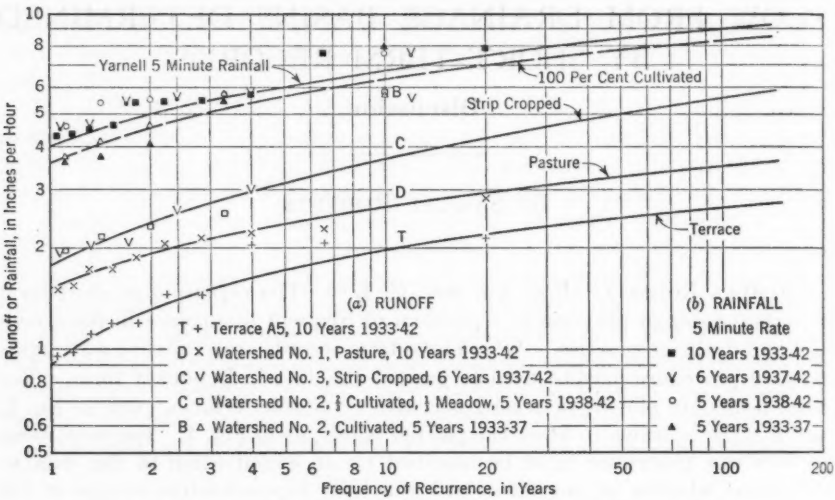

Runoff or Rainfall, in Inches per Hour
Frequency of Recurrence, in Years
Yarnell 5 Minute Rainfall
100 Per Cent Cultivated
Strip Cropped
Pasture
Terrace
(a) RUNOFF
T + Terrace A5, 10 Years 1933-42
D × Watershed No. 1, Pasture, 10 Years 1933-42
C V Watershed No. 3, Strip Cropped, 6 Years 1937-42
C □ Watershed No. 2, ½ Cultivated, ½ Meadow, 5 Years 1938-42
B △ Watershed No. 2, Cultivated, 5 Years 1933-37
(b) RAINFALL
5 Minute Rate
10 Years 1933-42
6 Years 1937-42
5 Years 1938-42
5 Years 1933-37

FIG. 8.—VALUES OF PEAK RUNOFF FOR VARIOUS RECURRENCE INTERVALS, EQ. 3

It is singular to note that the curves thus drawn seem to support the common practice of using the rational method and the Yarnell rainfall frequecy curve. Judicious choice of the runoff coefficient based on such information as that given by the author and the frequency curves in Fig. 8 would result in a more balanced design. The coefficient of runoff for area A-5 varies from 0.25 to 0.3. For area 1 it is about 0.4; and for area 3 and area 2 (between 1938 and 1942), it varies from 0.45 to 0.6. The coefficient for area 2 (between 1933 and 1937) is extraordinarily high and seems to indicate an inconsistency in the data, possibly caused by the changes in cultivation or plowing methods that may have reduced the concentration time; or perhaps the introduction of ground-water flow from other areas increased the flow measured at the gage. If the concentration time of area 2 had remained 5 min, it is improbable that the runoff rate would have exceeded the maximum 5-min rainfall rate for four out of five of the plotted values for the same period (see Fig. 8).

The areas studied by the author are so small that special cultivation methods may have an appreciable effect on the concentration time. From the descriptions given, the areas do not appear as dissimilar as the range in derived runoff coefficient indicates, and the assumption of a 5-min concentration time for each of the areas may not be valid. The data on hand are too inconsistent and variable to use for direct extrapolation to other watersheds. However, the paper can serve as a useful basis for further study, particularly if more information is published later for the same areas.